3. Repeat Exercise 1 using ~~Algorithm 6.2~~. Partial pivoting

c.
$$\begin{aligned} 2x_1 - 3x_2 + 2x_3 &= 5, \\ -4x_1 + 2x_2 - 6x_3 &= 14, \\ 2x_1 + 2x_2 + 4x_3 &= 8. \end{aligned}$$

$$\left[\begin{array}{ccc|c} 2 & -3 & 2 & 5 \\ -4 & 2 & -6 & 14 \\ 2 & 2 & 4 & 8 \end{array}\right] \xrightarrow{R1 \leftrightarrow R2} \left[\begin{array}{ccc|c} -4 & 2 & -6 & 14 \\ 2 & -3 & 2 & 5 \\ 2 & 2 & 4 & 8 \end{array}\right] \xrightarrow[\frac{1}{2}R_1+R_3]{\frac{1}{2}R_1+R_2} \left[\begin{array}{ccc|c} -4 & 2 & -6 & 14 \\ 0 & -2 & -1 & 12 \\ 0 & 3 & 1 & 15 \end{array}\right]$$

$\max\{|2|, |-4|, |2|\} = 4$

$\frac{a_{21}}{a_{11}} = \frac{2}{-4} = -\frac{1}{2}$, $\frac{a_{31}}{a_{11}} = \frac{2}{-4} = -\frac{1}{2}$

$\frac{1}{2}R_1 + R_2$ $\quad$ $\frac{1}{2}R_1 + R_3$

$\max\{|-2|, |3|\}$

$$\xrightarrow{R2 \leftrightarrow R3} \left[\begin{array}{ccc|c} -4 & 2 & -6 & 14 \\ 0 & 3 & 1 & 15 \\ 0 & -2 & -1 & 12 \end{array}\right]$$

$\frac{a_{32}}{a_{22}} = \frac{-2}{3}$ $\quad$ $\frac{2}{3}R_2 + R_3$

$$\xrightarrow{\frac{2}{3}R_2+R_3} \left[\begin{array}{ccc|c} -4 & 2 & -6 & 14 \\ 0 & 3 & 1 & 15 \\ 0 & 0 & -\frac{1}{3} & 22 \end{array}\right]$$

$-3 \times \quad -\frac{1}{3}x_3 = 22 \times -3$

$x_3 = -66$

$3x_2 + x_3 = 15$

$3x_2 - 66 = 15$

$+66 \quad +66$

$\frac{3x_2}{3} = \frac{81}{3}$

$x_2 = 27$

$-4x_1 + 2x_2 - 6x_3 = 14$

$-4x_1 + 2(27) - 6(-66) = 14$

$-4x_1 + 450 = 14$

$-450 \quad -450$

$\frac{-4x_1}{-4} = \frac{-436}{-4}$

$x_1 = 109$

5. Repeat Exercise 1 using ~~Algorithm 6.3~~. Scaled partial pivoting

c.
$$2x_1 - 3x_2 + 2x_3 = 5,$$
$$-4x_1 + 2x_2 - 6x_3 = 14,$$
$$2x_1 + 2x_2 + 4x_3 = 8.$$

$$\begin{bmatrix} \textcircled{2} & -3 & 2 & 5 \\ -4 & 2 & -6 & 14 \\ \textcircled{2} & 2 & 4 & 8 \end{bmatrix} \xrightarrow[-R_1+R_3]{2R_1+R_2} \begin{bmatrix} 2 & -3 & 2 & 5 \\ 0 & -4 & -2 & 24 \\ 0 & 5 & 2 & 3 \end{bmatrix} \xrightarrow{R2 \leftrightarrow R3} \begin{bmatrix} 2 & -3 & 2 & 5 \\ 0 & 5 & 2 & 3 \\ 0 & -4 & -2 & 24 \end{bmatrix}$$

$s_1 = \max\{|2|, |-3|, |2|\} = 3$

$s_2 = \max\{|-4|, |2|, |-6|\} = 6$

$s_3 = \max\{|2|, |4|, |2|\} = 4$

$\frac{|a_{11}|}{s_1} = \frac{2}{3}$, $\frac{|a_{21}|}{s_2} = \frac{4}{6} = \frac{2}{3}$, $\frac{|a_{31}|}{s_3} = \frac{2}{4} = \frac{1}{2}$

$\frac{a_{21}}{a_{11}} = \frac{-4}{2} = -2$, $\frac{a_{31}}{a_{11}} = \frac{2}{2} = 1$

$\frac{|a_{22}|}{s_2} = \frac{|-4|}{6} = \frac{2}{3}$ $\frac{|a_{32}|}{s_3} = \frac{5}{4}$

$\frac{a_{32}}{a_{22}} = \frac{-4}{5}$

$$\xrightarrow{\frac{4}{5}R_2+R_3} \begin{bmatrix} 2 & -3 & 2 & 5 \\ 0 & 5 & 2 & 3 \\ 0 & 0 & -\frac{2}{5} & \frac{132}{5} \end{bmatrix}$$

$-\frac{5}{2} \times -\frac{2}{5}x_3 = \frac{132}{5} \times -\frac{5}{2}$

$x_3 = -66$

$5x_2 + 2x_3 = 3$

$5x_2 + 2(-66) = 3$

$5x_2 - 132 = 3$

$+132 \quad +132$

$\frac{5x_2}{5} = \frac{135}{5}$

$x_2 = 27$

$2x_1 - 3x_2 + 2x_3 = 5$

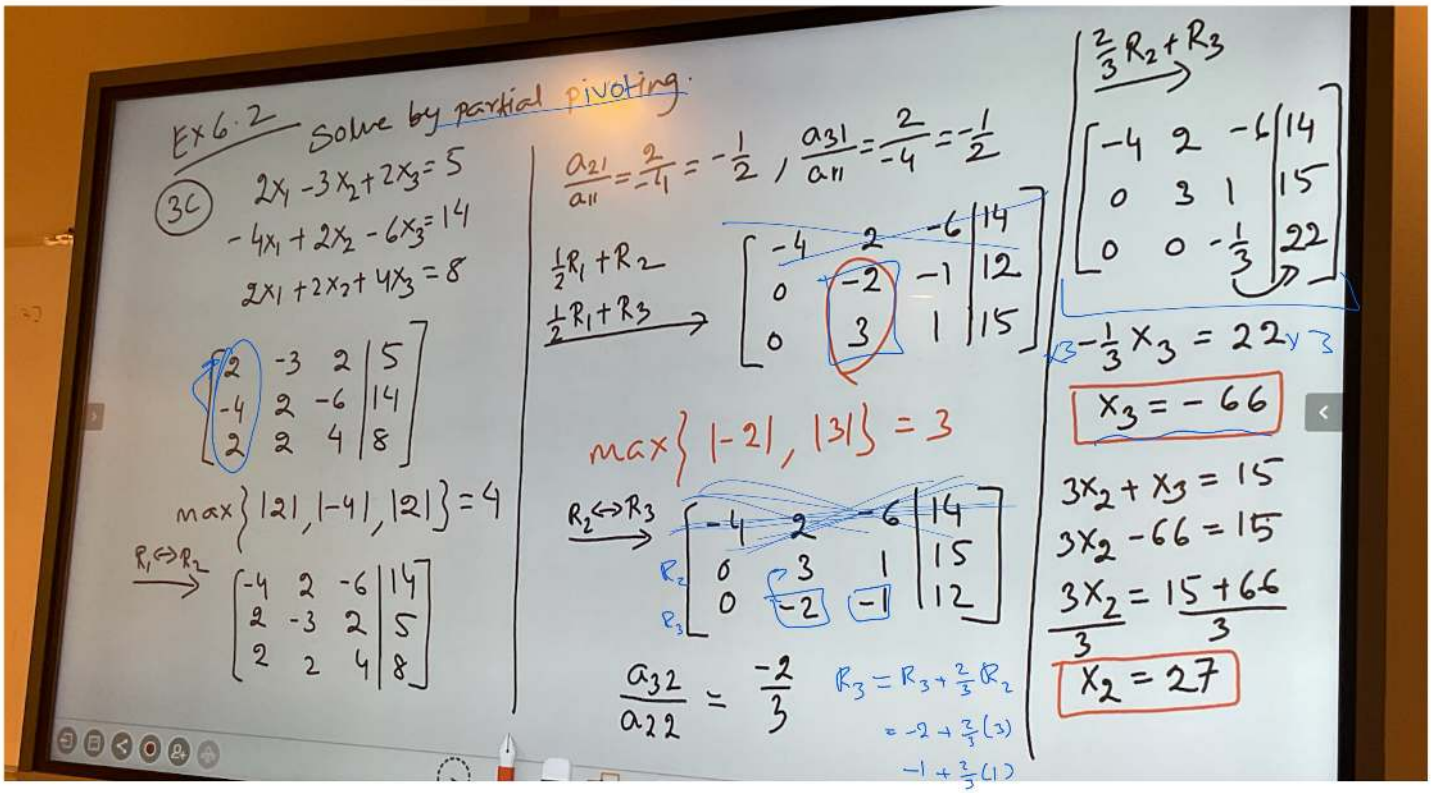
$2x_1 - 3(27) + 2(-66) = 5$

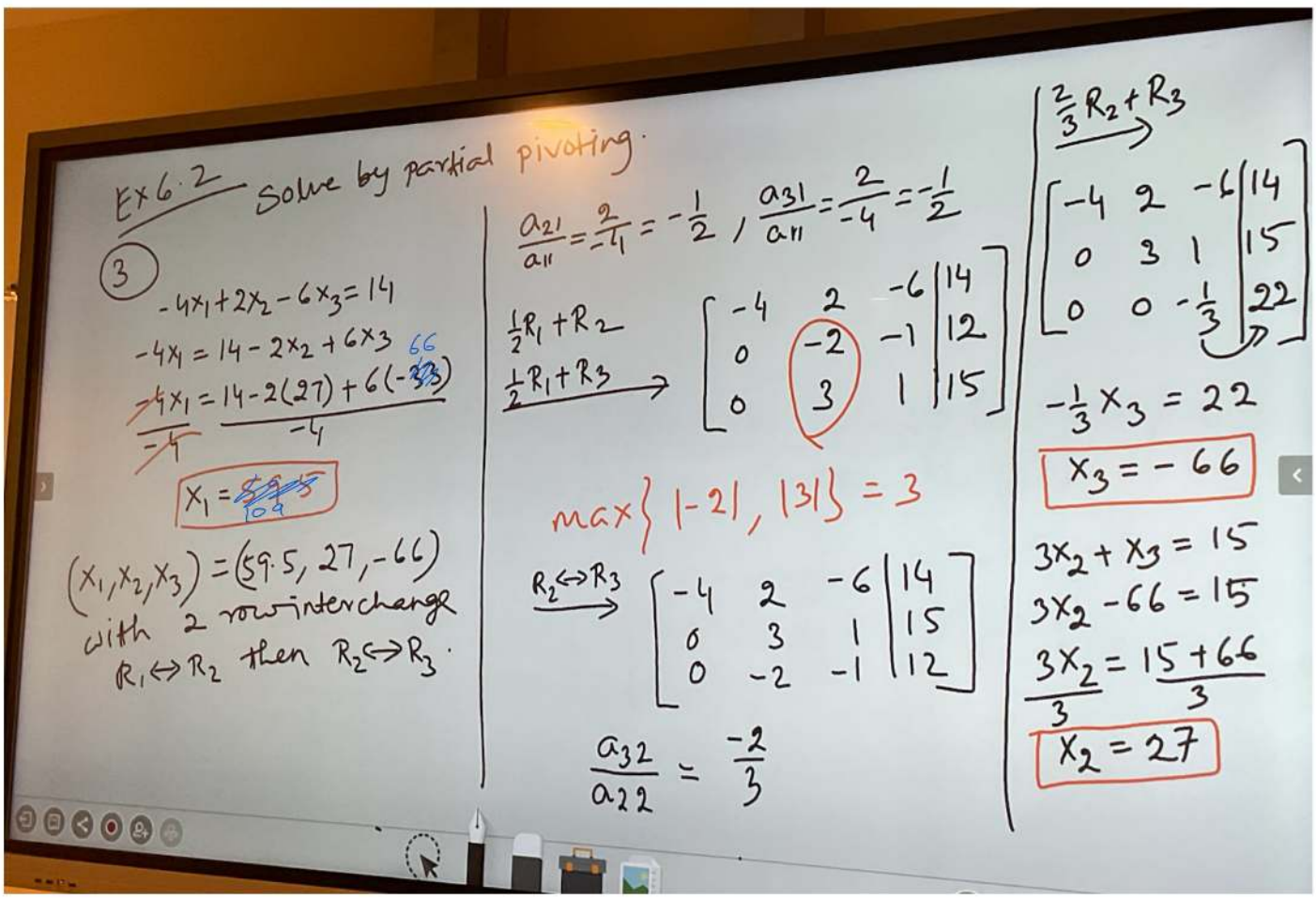
$2x_1 - 213 = 5$

$+213 \quad +213$

$2x_1 = 218$

$x_1 = 109$


Ex 6.2 Solve by partial pivoting.
(3c) $2x_1 - 3x_2 + 2x_3 = 5$
$-4x_1 + 2x_2 - 6x_3 = 14$
$2x_1 + 2x_2 + 4x_3 = 8$
$\max\{|2|, |-4|, |2|\} = 4$
$R_1 \leftrightarrow R_2$
$\frac{a_{21}}{a_{11}} = \frac{2}{-4} = -\frac{1}{2}$ / $\frac{a_{31}}{a_{11}} = \frac{2}{-4} = -\frac{1}{2}$
$\frac{1}{2}R_1 + R_2$
$\frac{1}{2}R_1 + R_3$
$\max\{|-2|, |3|\} = 3$
$R_2 \leftrightarrow R_3$
$\frac{a_{32}}{a_{22}} = \frac{-2}{3}$
$R_3 = R_3 + \frac{2}{3}R_2$
$= -2 + \frac{2}{3}(3)$
$-1 + \frac{2}{3}(1)$
$\frac{2}{3}R_2 + R_3$
$-\frac{1}{3}x_3 = 22$
$x_3 = -66$
$3x_2 + x_3 = 15$
$3x_2 - 66 = 15$
$\frac{3x_2}{3} = \frac{15+66}{3}$
$x_2 = 27$


Ex 6.2 Solve by partial pivoting.
(3) $-4x_1 + 2x_2 - 6x_3 = 14$
$-4x_1 = 14 - 2x_2 + 6x_3$
$-4x_1 = 14 - 2(27) + 6(-66)$
-4 -4
$x_1 = 59.5$
$(x_1, x_2, x_3) = (59.5, 27, -66)$
with 2 row interchange
$R_1 \leftrightarrow R_2$ then $R_2 \leftrightarrow R_3$.
$\frac{a_{21}}{a_{11}} = \frac{2}{-4} = -\frac{1}{2}$ / $\frac{a_{31}}{a_{11}} = \frac{2}{-4} = -\frac{1}{2}$
$\frac{1}{2}R_1 + R_2$
$\frac{1}{2}R_1 + R_3$
$\max\{|-2|, |3|\} = 3$
$R_2 \leftrightarrow R_3$
$\frac{a_{32}}{a_{22}} = \frac{-2}{3}$
$\frac{2}{3}R_2 + R_3$
$-\frac{1}{3}x_3 = 22$
$x_3 = -66$
$3x_2 + x_3 = 15$
$3x_2 - 66 = 15$
$\frac{3x_2}{3} = \frac{15+66}{3}$
$x_2 = 27$

(5c) Solve the system of linear eqns using scaled partial pivoting.

$$2x_1 - 3x_2 + 2x_3 = 5$$
$$-4x_1 + 2x_2 - 6x_3 = 14$$
$$2x_1 + 2x_2 + 4x_3 = 8$$

$$\begin{bmatrix} 2 & -3 & 2 & 5 \\ -4 & 2 & -6 & 14 \\ 2 & 2 & 4 & 8 \end{bmatrix}$$

$s_1 = \max\{|2|, |-3|, |2|\} = 3$

$s_2 = \max\{|-4|, |2|, |-6|\} = 6$

$s_3 = \max\{|2|, |2|, |4|\} = 4$

$\frac{|a_{11}|}{s_1} = \frac{2}{3}$, $\frac{|a_{21}|}{s_2} = \frac{4}{6} = \frac{2}{3}$, $\frac{|a_{31}|}{s_3} = \frac{2}{4} = \frac{1}{2}$

$\frac{a_{21}}{a_{11}} = \frac{-4}{2} = -2$, $\frac{2}{2} = 1 = \frac{a_{31}}{a_{11}}$

$2R_1 + R_2$, $-R_1 + R_3$

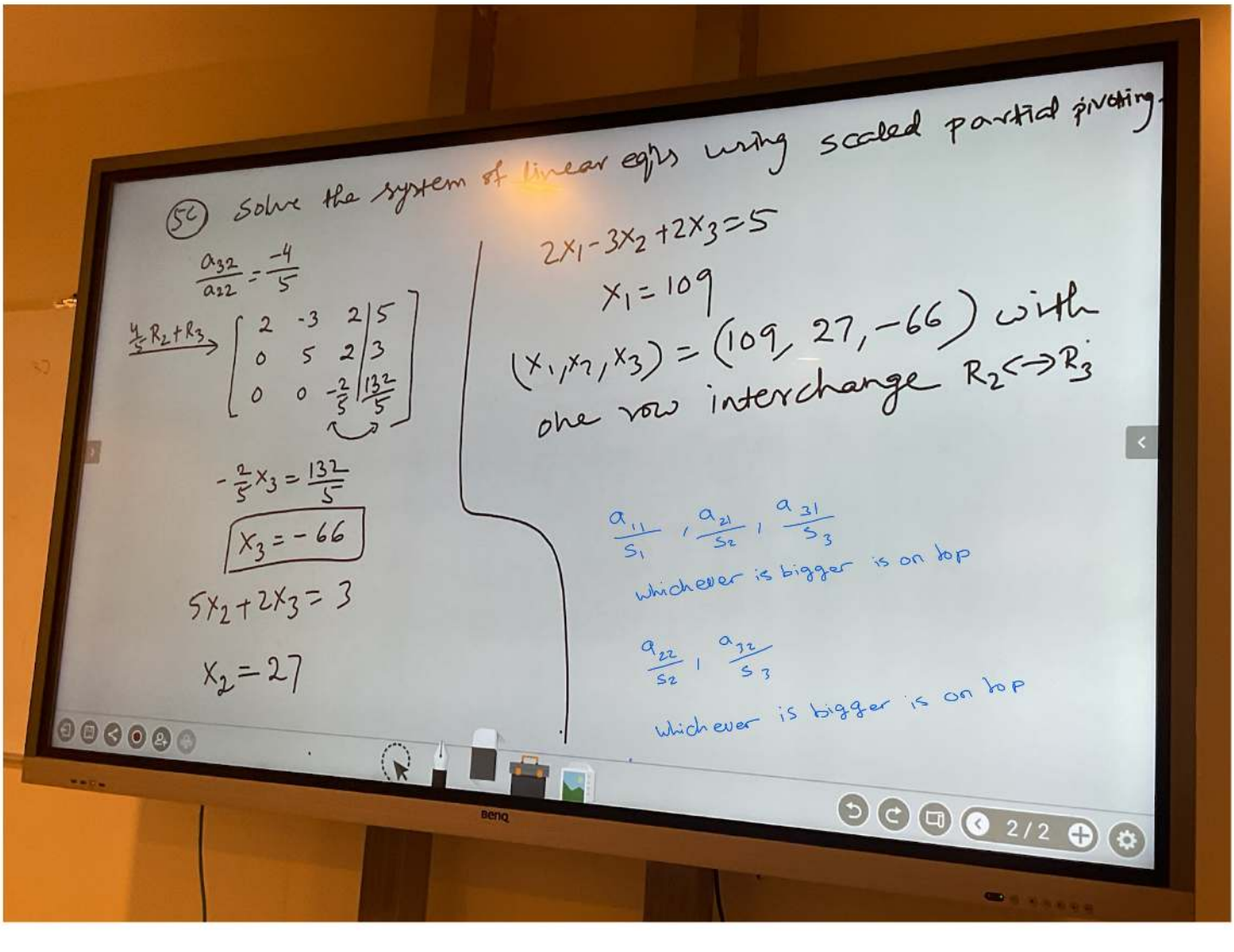
$$\begin{bmatrix} 2 & -3 & 2 & 5 \\ 0 & -4 & -2 & 24 \\ 0 & 5 & 2 & 3 \end{bmatrix}$$

$\frac{4}{6} = 0.66$, $\frac{5}{4} = 1.25$

$\frac{|a_{22}|}{s_2} = \frac{|-4|}{6} = \frac{2}{3}$, $\frac{|a_{32}|}{s_3} = \frac{5}{4}$

$R_2 \leftrightarrow R_3$

$$\begin{bmatrix} 2 & -3 & 2 & 5 \\ 0 & 5 & 2 & 3 \\ 0 & -4 & -2 & 24 \end{bmatrix}$$



Partial pivoting

$\max \{|a_{11}|, |a_{21}|, |a_{31}|\}$

Scaled partial pivoting

a_{11}

