Numerical differentiation

Approximation formulas

⇒ Forward - Endpoint ; $h>0$

$$f'(x_0) = \frac{f(x_0+h) - f(x_0)}{h}, \text{ where } h = x_1 - x_0 =$$

$$\Rightarrow \text{Error Bound} = \frac{h}{2} f''(\xi)$$

⇒ Backward - Endpoint ; $h<0$

$$f(x_0) = \frac{f(x_0+(-h)) - f(x_0)}{-h}, \text{ h should be -ve}$$

$$\Rightarrow \text{Error Bound} = \frac{h}{2} f''(\xi)$$

⇒ Three point formulas ; *Can use forward n| Backward &

Three point - Endpoint ⇒ $f(x_0) = \frac{1}{2h}\left[-3f(x_0) + 4f(x_0+h) - f(x_0+2h)\right]$

$$\text{Error. B} = \frac{h^2}{3} f'''(\xi)$$

Three point - Midpoint ⇒ $f(x_0) = \frac{1}{2h}\left[f(x_0+h) - f(x_0-h)\right]$

$$, \text{Error. B} = \frac{h^2}{6} f'''(\xi_1) \text{ using } x_0-h \text{ and } x_0+h$$

⇒ Five point formulas ; *Can use forward n| Backward &

Five point - Endpoint ⇒ $f(x_0) = \frac{1}{12h}\left[-25f(x_0) + 48f(x_0+h) - 36f(x_0+2h) + 16f(x_0+3h) - 3f(x_0+4h)\right]$

$$\text{Error. B} = \frac{h^4}{5} f^5(\xi)$$

Five point - Mid point $f(x_0) = \frac{1}{12h}\left[f(x_0-2h) - 8f(x_0-h) + 8f(x_0+h) - f(x_0+2h)\right]$

$$\text{Error. B} = \frac{h^4}{30} f^5(\xi)$$

Q1 %

1. Use the forward-difference formulas and backward-difference formulas to determine each missing entry in the following tables.

a.

x	$f(x)$	$f'(x)$
0.5	0.4794	0.852 → f
0.6	0.5646	0.852 → B
0.7	0.6442	0.796 → B

$h = 0.1$

forward % $h = 0.1$.

$f(0.5) = \frac{f(0.6) - f(0.5)}{0.1} = \boxed{0.852}$

$f(0.6) = \frac{f(0.7) - f(0.6)}{0.1} = \boxed{0.796}$

$f(0.7) = \frac{f(0.8) - f(0.7)}{0.1}$ = Can't be solved bc. 0.4 not in table.

Backword % $h = -0.1$

$f(0.5) = \frac{f(0.5+(-0.1)) - f(0.5)}{-0.1} = \frac{f(0.4) - f(0.5}{-0.1}$ = Can't be solved bc. 0.4 not in table.

$f(0.6) = \frac{f(0.5) - f(0.6)}{-0.1} = \boxed{0.852}$

$f(0.7) = \frac{f(0.6) - f(0.7)}{-0.1} = \boxed{0.796}$

Q3 %

3. The data in Exercise 1 were taken from the following functions. Compute the actual errors in Exercise 1, and find error bounds using the error formulas.

a. $f(x) = \sin x$

① find $f'(x)$ and $f''(x)$ %

$f'(x) = \cos x$

$f''(x) = -\sin x$ ('ll use for E.Bound)

② Actual. Error % $|$ Actual. E − Apper. E $|$ ← from Q1

① $\Rightarrow f'(0.5) = \cos(0.5) = 0.8775$

$f'(0.6) = 0.8253$

$f'(0.7) = 0.7648$

② A.E for $(0.5) = |0.852 - 0.8775| = \underline{0.0255}$

$(0.6) = |0.852 - 0.8253| = \underline{0.0267}$

$(0.7) = |0.796 - 0.7648| = \underline{0.0312}$

③ final E. Bound & $\left|\frac{h}{2} f''(\xi)\right|$

for 0.5 $[0.5, 0.6] = \left|\frac{0.1}{2}(-\sin(\xi))\right| = |0.05(-\sin(\xi))| = |-0.05 \times 0.5646| = \underline{0.02823}$

$0.5 \leq \sin(\xi) \leq 0.6$ ← Substituent 0.6 and 0.5 in Sin

0.4794 ↓ 0.5646 (boxed)

for 0.6 $[0.6, 0.7] = |-0.05 \times 0.6442| = \underline{0.03221}$

$0.6 \leq \sin(\xi) \leq 0.7$

0.5646 ↓ 0.6442 (boxed)

for 0.7 $[0.6, 0.7] = \underline{0.03221}$

Q58

5. Use the most accurate three-point formula to determine each missing entry in the following tables.

d.

	x	$f(x)$	$f'(x)$
forwards ←	x_0	2.0	3.6887983	
mid-Point ←	x_1	2.1	3.6905701	
	x_2	2.2	3.6688192	
backwards ←	x_3	2.3	3.6245909	

$f'(x_0) = \frac{1}{2h}[-3f(x_0) + 4f(x_0+h) - f(x_0+2h)]$

Error. $B = \frac{h^2}{3} f'''(\xi)$

Three point – Midpoint $\Rightarrow f'(x_0) = \frac{1}{2h}[f(x_0+h) - f(x_0-h)]$

, Error. $B = \frac{h^2}{6} f'''(\xi_1)$ using x_0+h and x_0+h

$h = 0.1$

Three-point – Endpoint's

Endpoint (forward)'s $h>0$

$f'(x_0) = \frac{1}{2h}[-3f(x_0) + 4f(x_0+h) - f(x_0+2h)] = 0.1353$

$f'(x_1) = -0.1051$

Endpoint (Backward)'s $h<0 \Rightarrow -0.1$

$f'(2.2) = \frac{1}{2(-0.1)}[-3f(x_2) + 4f(x_2+h) - f(x_2+2h)]$ (2.2, 2.1, 2.0)

$= -0.3351$

$f'(2.3) = -0.5546$

Three-point – Midpoint's → must use mid point only "x_1, x_2"

$f'(x_1) = \frac{1}{2h}[f(x_1+h) - f(x_1-h)] = -0.09989$ (2.2, 2.1)

$f'(x_2) = -0.329896$