Divied Difference

General formulas

$x_0 \to f(x_0)$

$x_1 \to f(x_1)$ $\quad f(x_0, x_1) = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$

$x_2 \to f(x_2)$ $\quad f(x_1, x_2) = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$ $\quad f(x_0, x_1, x_2) = \frac{f(x_1, x_2) - f(x_0, x_1)}{x_2 - x_0}$

$x_3 \to f(x_3)$ $\quad f(x_2, x_3) = \frac{f(x_3) - f(x_2)}{x_3 - x_2}$ $\quad f(x_1, x_2, x_3) = \frac{f(x_2, x_3) - f(x_2 - x_1)}{x_3 - x_1}$ $\quad f(x_0, x_1, x_2, x_3) = \frac{f(x_1, x_2, x_3) - f(x_0, x_1, x_2)}{x_3 - x_0}$

$\Rightarrow$ Polynomeil s "underlineel"

deg0 deg1 deg2 deg3

$$P_n(x) = f(x_0) + f(x_0, x_1)(x - x_0) + f(x_0, x_1, x_2)(x - x_0)(x - x_1) + f(x_0, x_1, x_2, x_3)(x - x_0)(x - x_1)(x - x_2) + \cdots$$

Q2 0

2. Use Eq. (3.10) or Algorithm 3.2 to construct interpolating polynomials of degree one, two, and three for the following data. Approximate the specified value using each of the polynomials.

a. $f(0.43)$ if $f(0) = 1$, $f(0.25) = 1.64872$, $f(0.5) = 2.71828$, $f(0.75) = 4.48169$

$x \Rightarrow 0.43$

$x_0 = 0$

$x_1 = 0.25$ $\Rightarrow$

$x_2 = 0.5$

$x_3 = 0.75$

$= \frac{f(x_1) - f(x_0)}{x_1 - x}$

$f(x_0, x_1) = 2.59488$

$f(x_1, x_2) = 4.27824$

$f(x_2, x_3) = 7.05364$

$= \frac{f(x_1, x_2) - f(x_0, x_1)}{x_2 - x_0}$

$f(x_0, x_1, x_2) = 3.66672$

$f(x_1, x_2, x_3) = 5.5508$

$= \frac{f(x_0, x_1, x_2) - f(x_1, x_2, x_3)}{x_3 - x_0}$

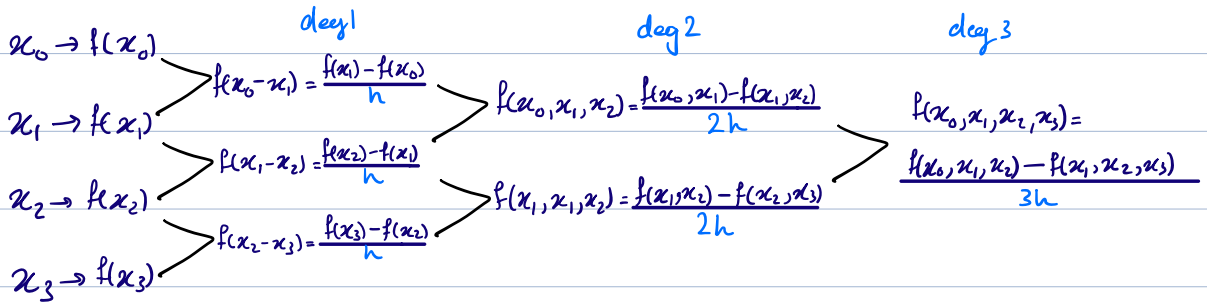
$f(x_0, x_1, x_2, x_3) = 2.5121066$

$P_1(0.43) = f(x_0) + f(x_0, x_1)(x - x_0) = 2.1157984$

$P_2(0.43) = P_1(x) + f(x_0, x_1, x_2)(x - x_0)(x - x_1) = 2.3996025$

$P_3(0.43) = P_2(x) + f(x_0, x_1, x_2, x_3)(x - x_0)(x - x_1)(x - x_2) = 2.3859919$

Netown's forward method :



$\Rightarrow h = x_1 - x_0 = x_2 - x_1 = x_3 - x_2$

$\Rightarrow s = \frac{x - x_0}{h}$, In forward method "s" is +ve

$\Rightarrow$ polynomials :

$$P_n(x) = f(x_0) + f(x_0,x_1)\,s\cdot h + f(x_0,x_1,x_2)\,s(s-1)h^2 + f(x_0,x_1,x_2,x_3)\,s(s-2)h^3 + \cdots$$

Q4 :

4. Use the Newton forward-difference formula to construct interpolating polynomials of degree one, two, and three for the following data. Approximate the specified value using each of the polynomials.

b. $f(0.18)$ if $f(0.1) = -0.29004986$, $f(0.2) = -0.56079734$, $f(0.3) = -0.81401972$, $f(0.4) = -1.0526302$

(x_0, x_1, x_2, x_3)

$x \Rightarrow 0.18$

$x_0 = 0.1$

$x_1 = 0.2$

$x_2 = 0.3$

$x_3 = 0.4$

$f(x_0,x_1) = -0.3384343$

$f(x_1,x_2) = -0.3165279$

$f(x_2,x_3) = -0.2982631$

$f(x_0,x_1,x_2) = 0.0136915$

$f(x_1,x_2,x_3) = 0.0114155$

$f(x_0,x_1,x_2,x_3) = -0.0009483$

$h = 0.1$

$s = \frac{0.18-0.1}{0.1} = 0.8$

$P(0.18) = f(x_0) + f(x_0,x_1)\,s\cdot h + f(x_0,x_1,x_2)\,s(s-1)h^2 + f(x_0,x_1,x_2,x_3)\,s(s-1)h^3$

$= -0.3171464$

Q9 **9. a.** Approximate $f(0.05)$ using the following data and the Newton forward-difference formula:

	x_0	x_1	x_2	x_3	x_4
x	0.0	0.2	0.4	0.6	0.8
$f(x)$	1.00000	1.22140	1.49182	1.82212	2.22554

$x \Rightarrow 0.05$

$x_0 = 0$

$x_1 = 0.2$ → $f(x_0, x_1) = 1.107$ → $f(x_0, x_1, x_2) = 0.61275$ → $f(x_0, x_1, x_2, x_3) = 0.22625$ → $f(x_0, x_1, x_2, x_3, x_4) = 0.040104166625$

$x_2 = 0.4$ → $f(x_1, x_2) = 1.3521$ → $f(x_1, x_2, x_3) = 0.7485$ → $f(x_1, x_2, x_3, x_4) = 0.2758333333$

$x_3 = 0.6$ → $f(x_2, x_3) = 1.6515$ → $f(x_2, x_3, x_4) = 0.914$

$x_4 = 0.8$ → $f(x_3, x_4) = 2.0171$

$h = 0.2$

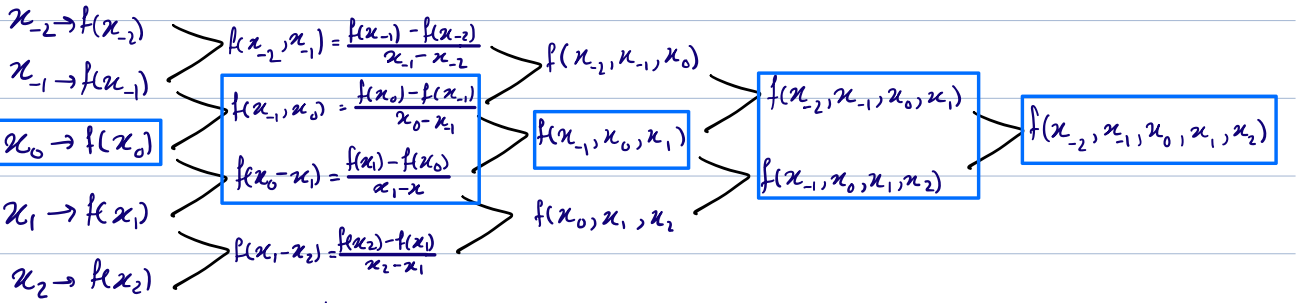
$s = \frac{0.5 - 0}{0.2} = 2.5$

$P(0.05) = f(x_0) + sh(f(x_0, x_1)) + s(s-1)h^2(f(x_0, x_1, x_2)) + s(s-1)h^3(f(x_0, x_1, x_2, x_3) + s(s-1)h^4($

$f(x_0, x_1, x_2, x_3, x_4) =$

$= 1.6524406$

Centered difference (Stirling formula) 8



$\Rightarrow h = x_1 - x_0 = x_2 - x_1 = x_3 - x_2$

$\Rightarrow s = \frac{x - x_0}{h}$, In Centeral method "s" can be +ve and -ve

$\Rightarrow$ polynomials

$$P_n(x) = f(x_0) + \frac{s \cdot h}{2}\left[f(x_{-1}, x_0) + f(x_0, x_1)\right] + s^2 h^2 \left[f(x_{-1}, x_0, x_1)\right] + \frac{s(s^2-1)}{2} \cdot h^3 \left[f(x_{-2}, x_{-1}, x_0, x_1) + f(x_{-1}, x_0, x_1, x_2)\right]$$

$$+ s^2(s^2-1)h^4 \left[f(x_{-2}, x_{-1}, x_0, x_1, x_2)\right] + \cdots$$

above $x_0 \Rightarrow x_{-1}, x_{-2}$ ....

below $x_0 \Rightarrow x_1, x_2$ ----

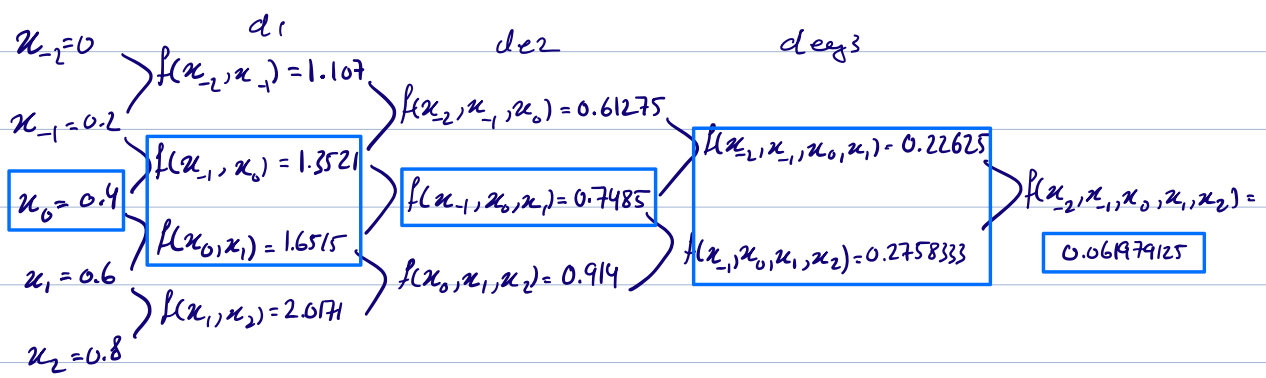
Q 9 0

c. Use Stirling's formula to approximate $f(0.43)$.

x	0.0	0.2	0.4	0.6	0.8
$f(x)$	1.00000	1.22140	1.49182	1.82212	2.22554

$h = 0.2$

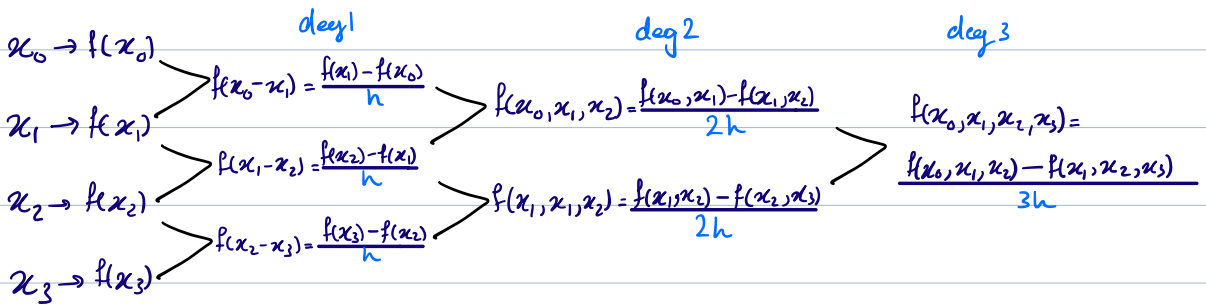
$s = \frac{x - x_0}{h} = \frac{0.43 - 0.4}{0.2} = .15$



$$P(0.43) = f(x_0) + \frac{sh}{2}\left(f(x_{-1},x_0) + f(x_0,x_1)\right) + s^2h^2 f(x_{-1},x_0,x_1) + \frac{s(s^2-1)h^3}{2}\left(f(x_{-1},x_0,x_1)\right) +$$

$$s^2(s^2-1)h^4\left(f(x_{-2},x_{-1},x_0,x_1,x_2)\right) = 1.537229\overline{13} \rightarrow \text{not scare}$$

Newton's Backword method



$\Rightarrow$ polynomials

$$P_n(x) = f(x_0) + \frac{s \cdot h}{2}\left[f(x_{-1}, x_0) + f(x_0, x_1)\right] + s^2 h^2 \left[f(x_{-1}, x_0, x_1)\right] + \frac{s(s^2-1)}{2} \cdot h^3 \left[f(x_{-2}, x_{-1}, x_0, x_1) + f(x_{-1}, x_0, x_1, x_2)\right]$$

$$+ s^2(s^2-1)h^4 \left[f(x_{-2}, x_{-1}, x_0, x_1, x_2)\right] + \cdots$$

above $x_0 \Rightarrow x_{-1}, x_{-2} \cdots$

below $x_0 \Rightarrow x_1, x_2 \cdots$

$\Rightarrow h = x_1 - x_0 = x_2 - x_1 = x_3 - x_2$

$\Rightarrow s = \frac{x - x_0}{h}$, In Backword method "s" is -ve

9. a. $f(0.05) \approx 1.05126$ b. $f(0.551) \approx 1.91555$ c. $f(0.43) \approx 1.53725$