

http://algs4.cs.princeton.edu

4.1 UNDIRECTED GRAPHS

- *introduction*
- *graph API*
- *depth-first search*
- *breadth-first search*
- *connected components*
- *challenges*



http://algs4.cs.princeton.edu

4.1 Undirected Graphs

edge → يصير دبل

ما يكون فيه اتجاهات → يحسب من الطرفين راديح وجاي

- *introduction*
- *graph API*
- *depth-first search*
- *breadth-first search*
- *connected components*
- *challenges*

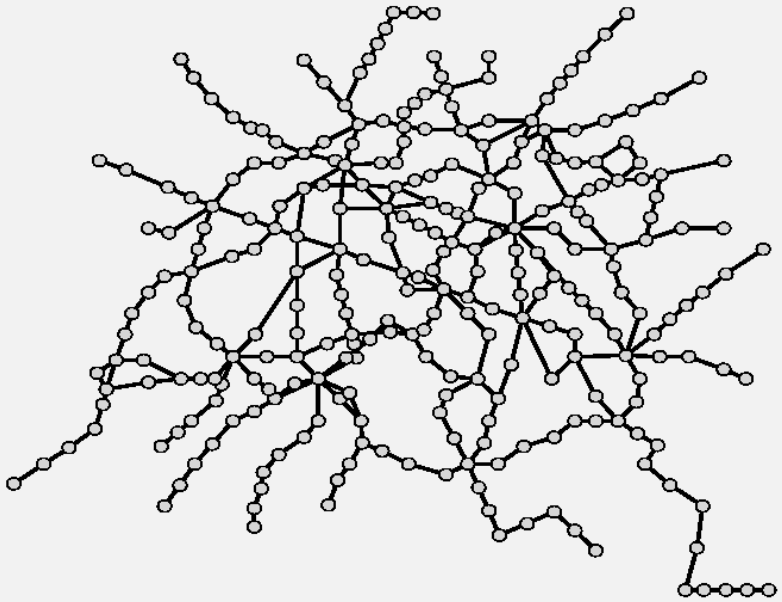
Undirected graphs

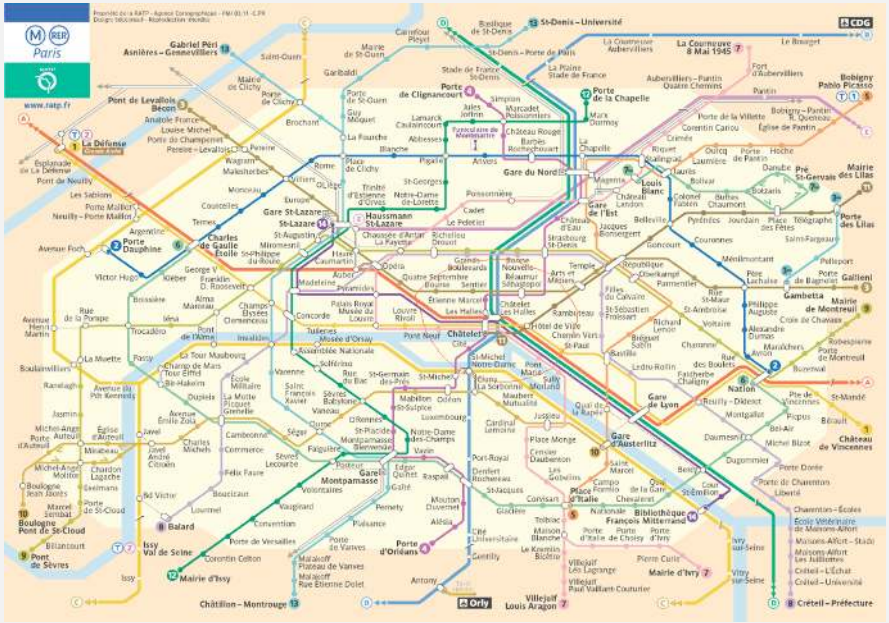
Graph. Set of vertices connected pairwise by edges.

تحسب من الدهتين

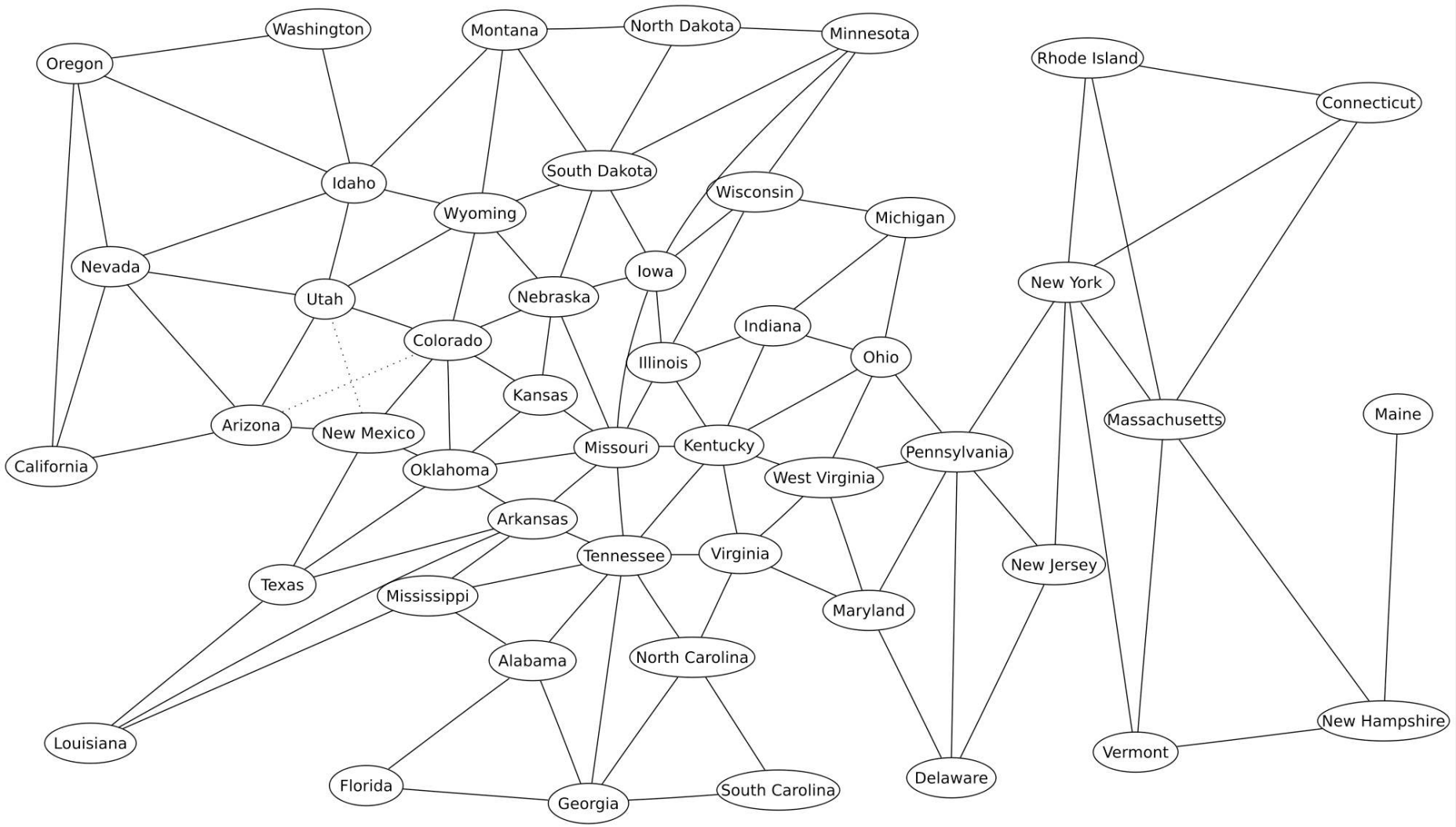
Why study graph algorithms?

- Thousands of practical applications.
- Hundreds of graph algorithms known.
- Interesting and broadly useful abstraction.
- Challenging branch of computer science and discrete math.

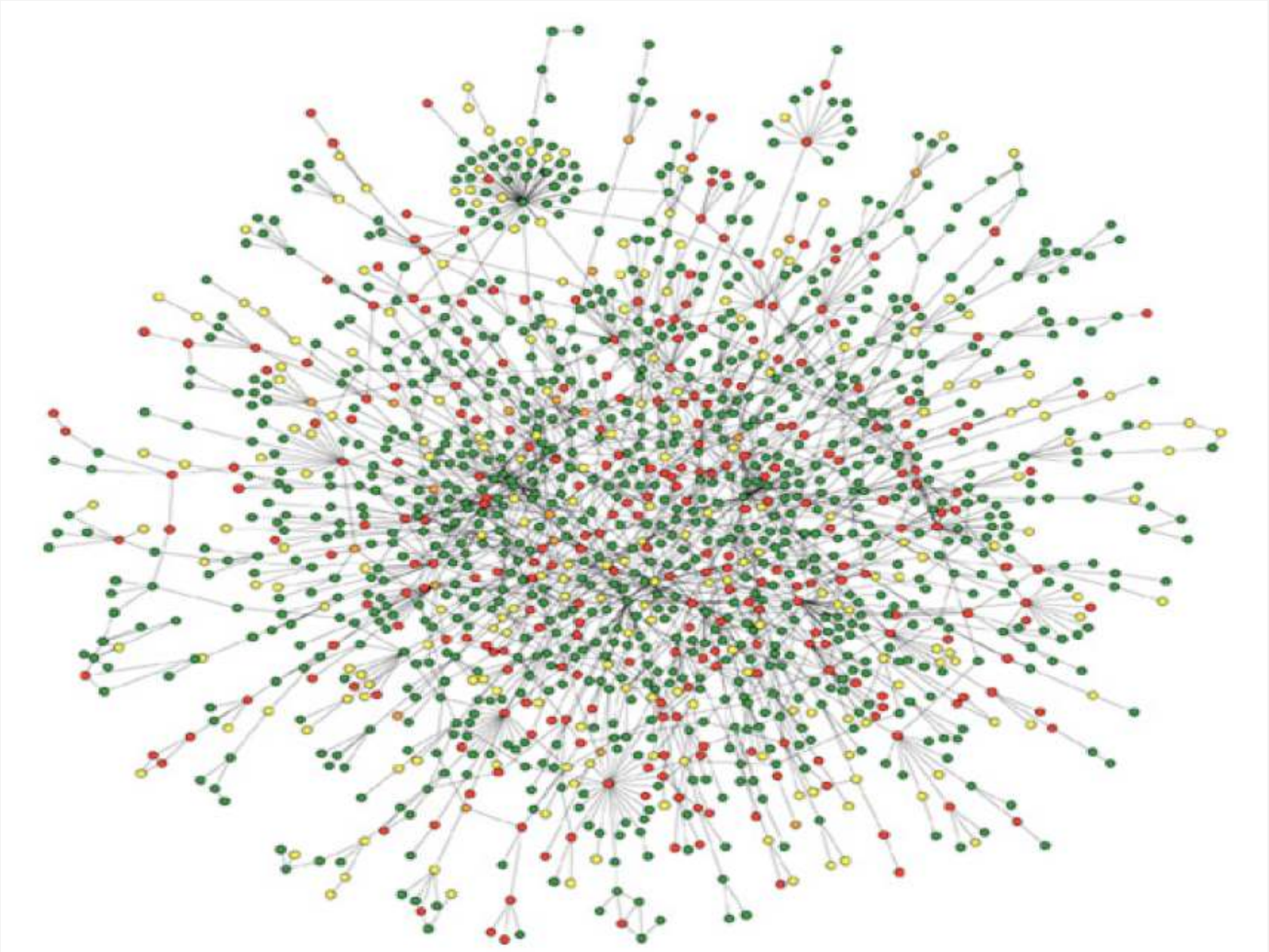




Border graph of 48 contiguous United States

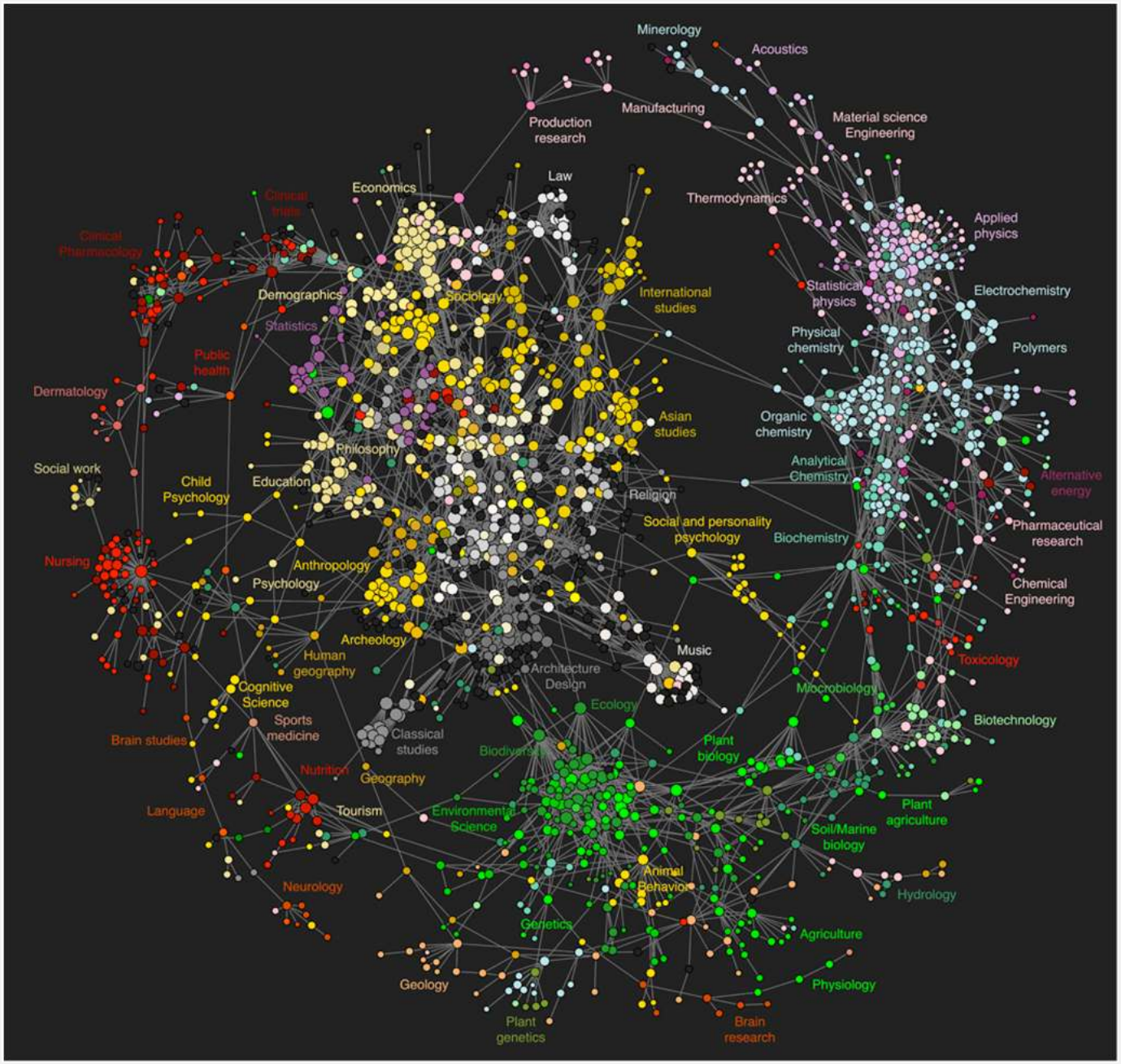


Protein-protein interaction network



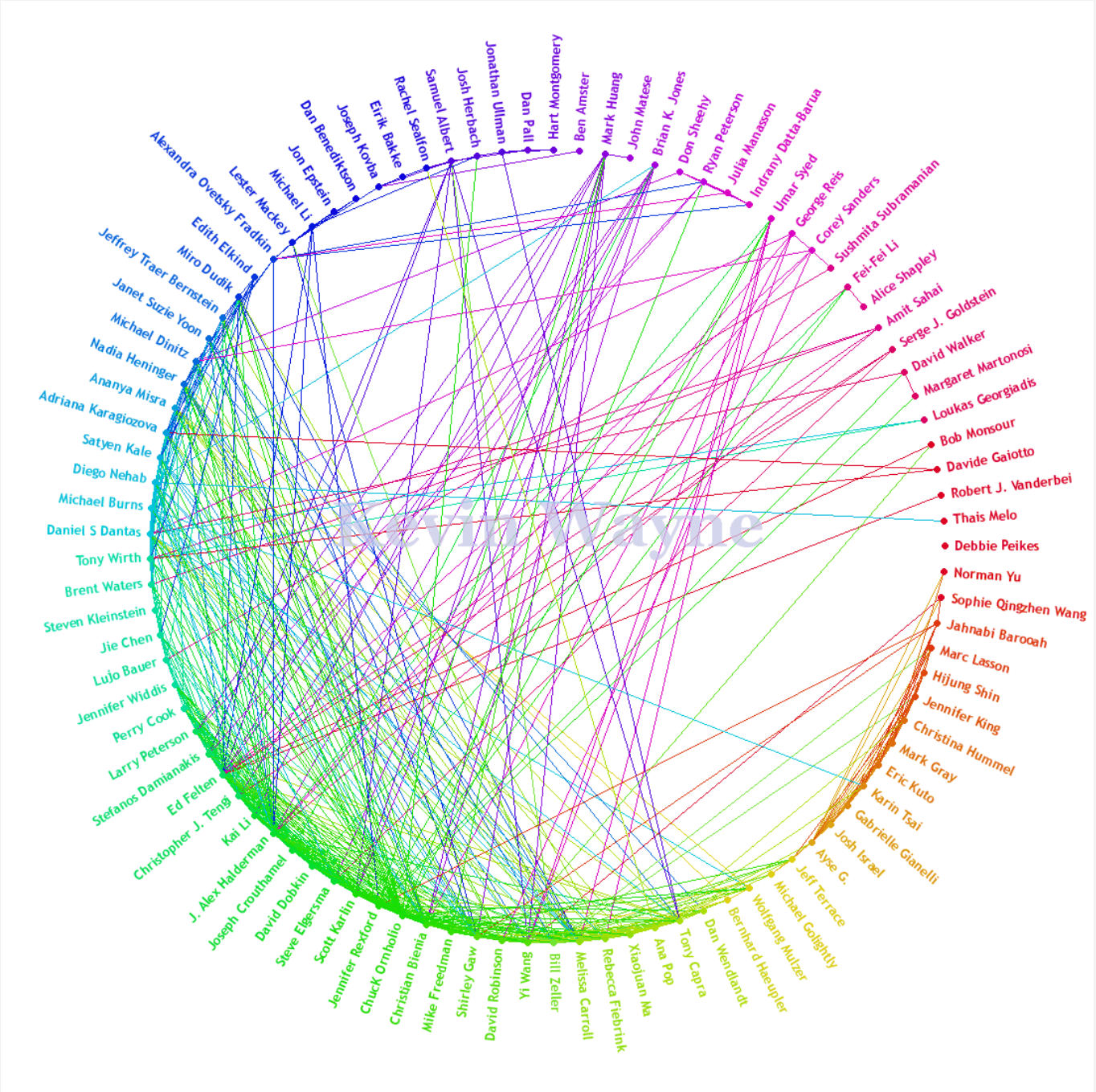
Reference: Jeong et al, Nature Review | Genetics

Map of science clickstreams



http://www.plosone.org/article/info:doi/10.1371/journal.pone.0004803

Kevin's facebook friends (Princeton network, circa 2005)

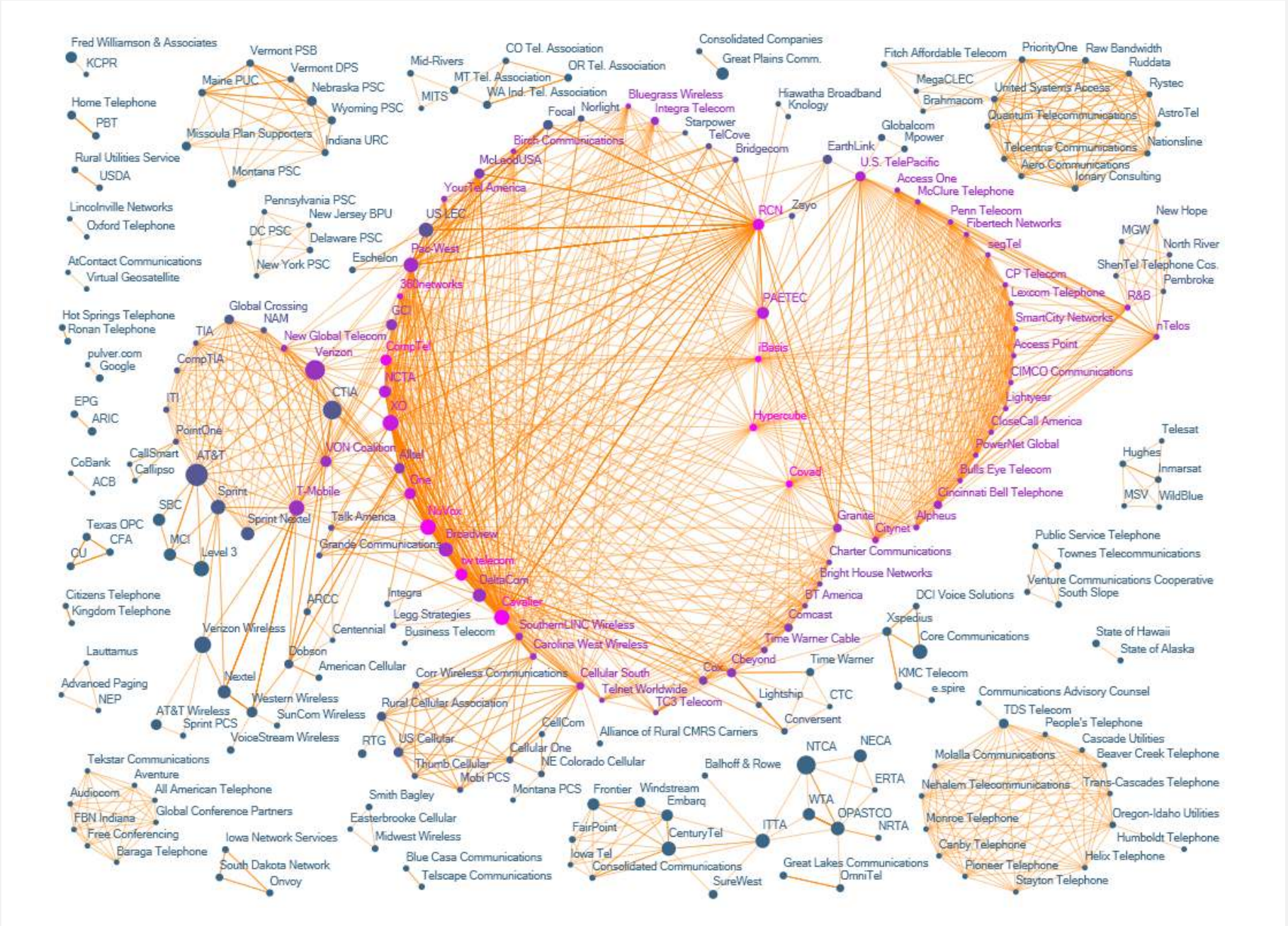


10 million Facebook friends



"Visualizing Friendships" by Paul Butler

The evolution of FCC lobbying coalitions



"The Evolution of FCC Lobbying Coalitions" by Pierre de Vries in JoSS Visualization Symposium 2010

Framingham heart study

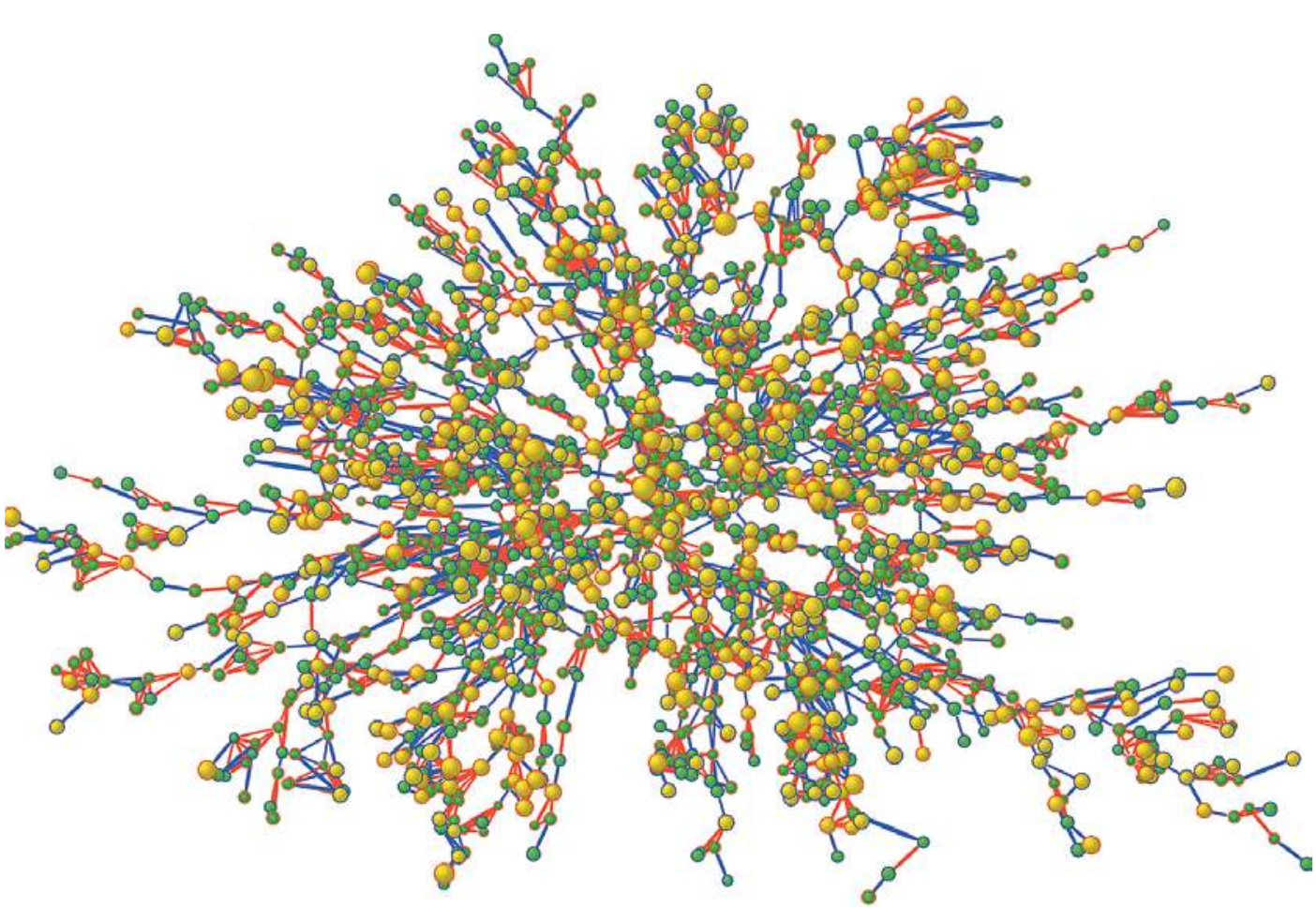
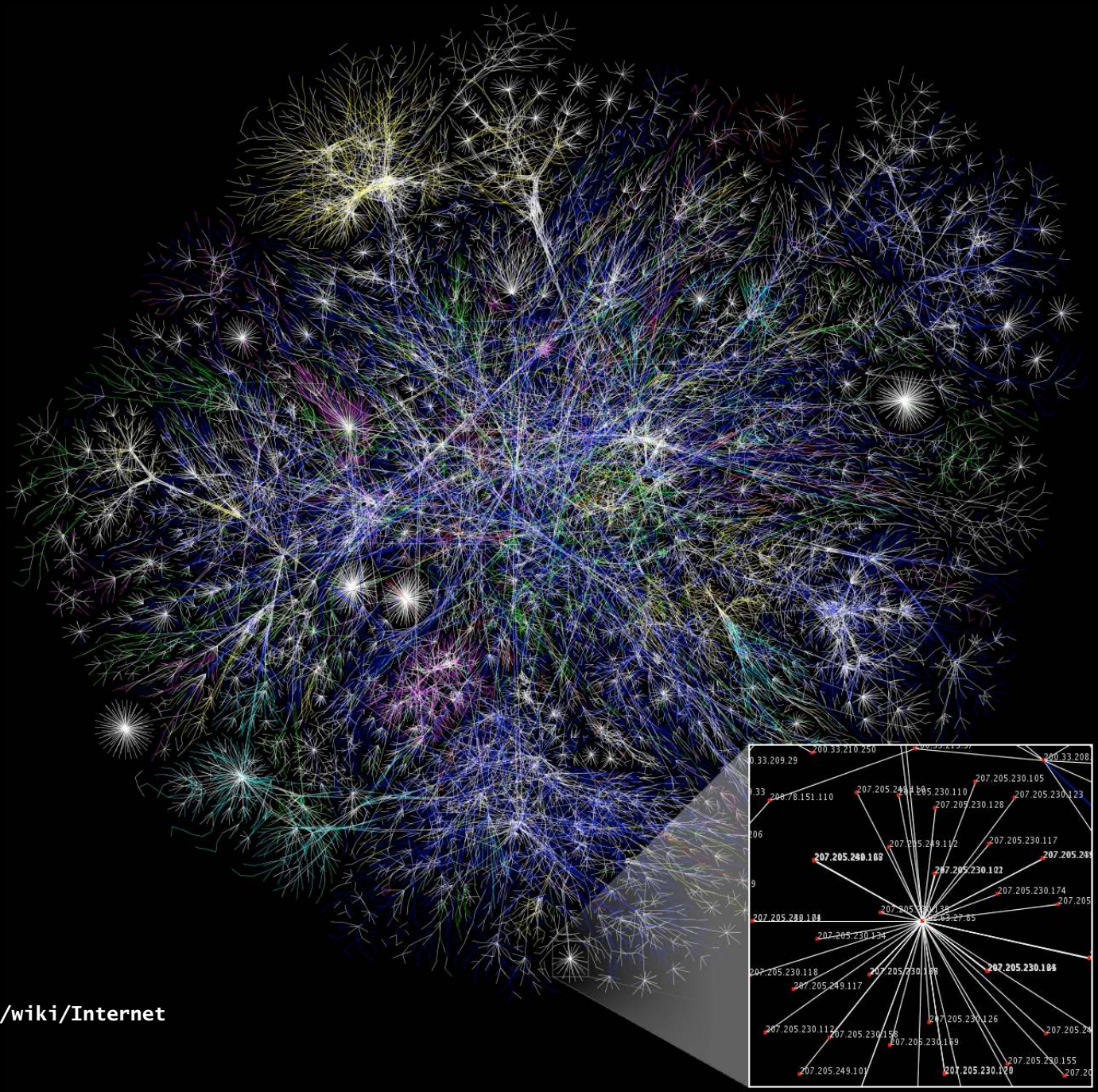


Figure 1. Largest Connected Subcomponent of the Social Network in the Framingham Heart Study in the Year 2000.

Each circle (node) represents one person in the data set. There are 2200 persons in this subcomponent of the social network. Circles with red borders denote women, and circles with blue borders denote men. The size of each circle is proportional to the person's body-mass index. The interior color of the circles indicates the person's obesity status: yellow denotes an obese person (body-mass index, ≥30) and green denotes a nonobese person. The colors of the ties between the nodes indicate the relationship between them: purple denotes a friendship or marital tie and orange denotes a familial tie.

"The Spread of Obesity in a Large Social Network over 32 Years" by Christakis and Fowler in New England Journal of Medicine, 2007

The Internet as mapped by the Opte Project



http://en.wikipedia.org/wiki/Internet

Graph applications

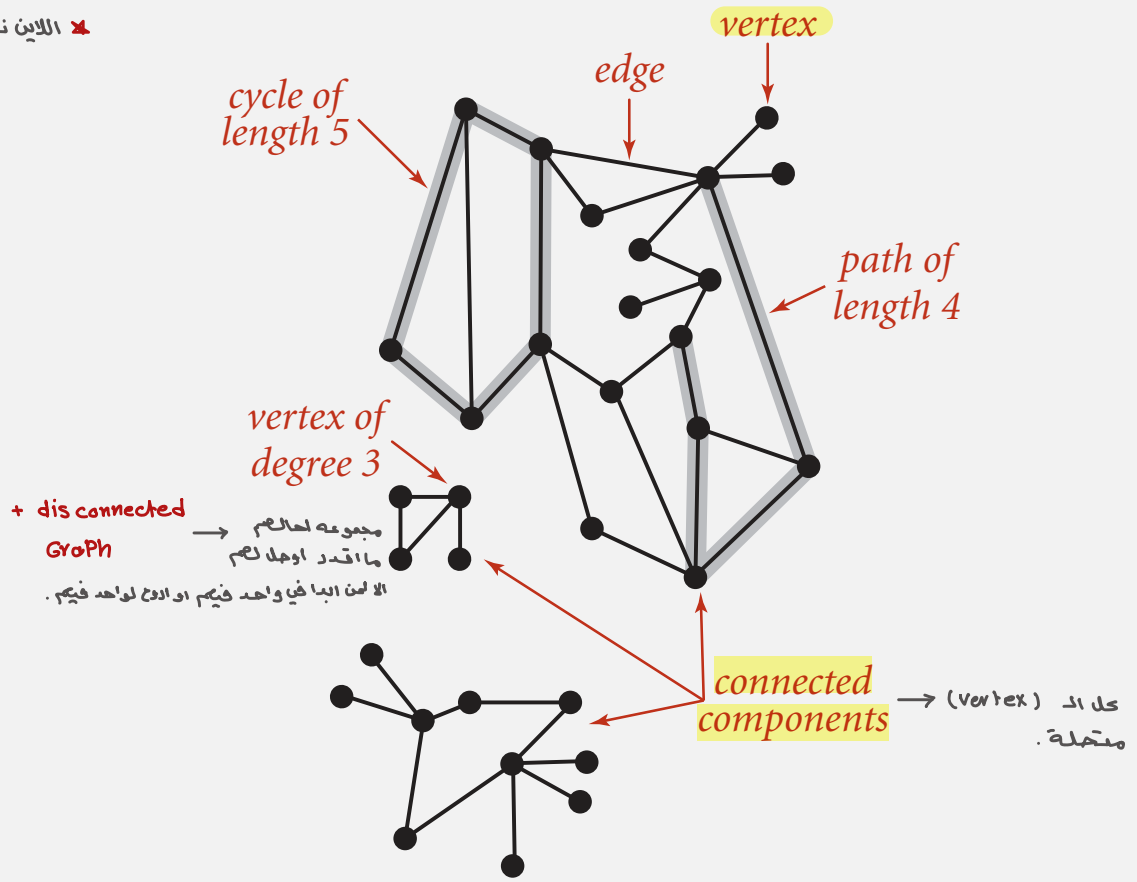
graph	vertex	edge
communication	telephone, computer	fiber optic cable
circuit	gate, register, processor	wire
mechanical	joint	rod, beam, spring
financial	stock, currency	transactions
transportation	intersection	street
internet	class C network	connection
game	board position	legal move
social relationship	person	friendship
neural network	neuron	synapse
protein network	protein	protein-protein interaction
molecule	atom	bond

Graph terminology

Path. Sequence of vertices connected by edges.

Cycle. Path whose first and last vertices are the same.

Two vertices are connected if there is a path between them.



Some graph-processing problems

problem	description
s-t path	*Is there a path between s and t ?*
shortest s-t path	*What is the shortest path between s and t ?*
cycle	*Is there a cycle in the graph ?*
Euler cycle	*Is there a cycle that uses each edge exactly once ?*
Hamilton cycle	*Is there a cycle that uses each vertex exactly once ?*
connectivity	*Is there a way to connect all of the vertices ?*
biconnectivity	*Is there a vertex whose removal disconnects the graph ?*
planarity	*Can the graph be drawn in the plane with no crossing edges ?*
graph isomorphism	*Do two adjacency lists represent the same graph ?*

Challenge. Which graph problems are easy? difficult? intractable?



http://algs4.cs.princeton.edu

4.1 Undirected Graphs

مافيها مؤشر وفيها (digraph). العلاقه رايحه باية.

- *introduction*
- *graph API*
- *depth-first search*
- *breadth-first search*
- *connected components*
- *challenges*

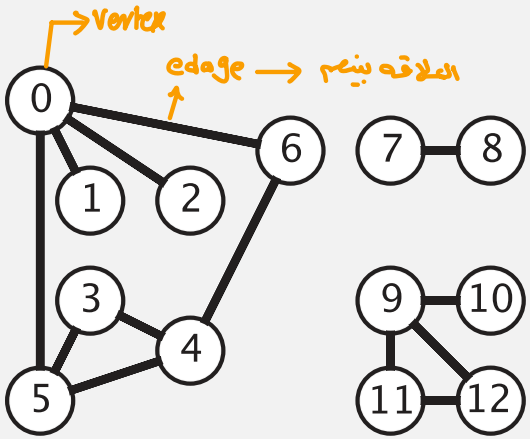
Graph representation

Graph drawing. Provides intuition about the structure of the graph.

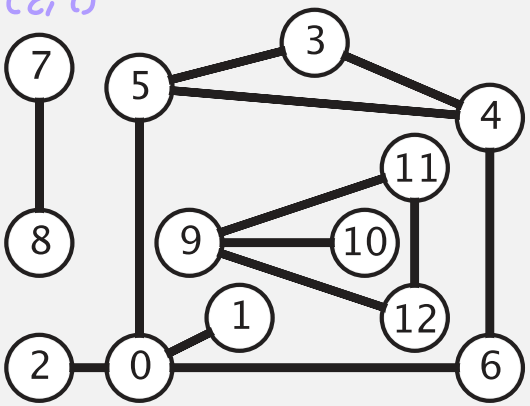
ماعندي مشكله بطريقه الرسم
اهم شي . اللاين ماانساها وال (Vertex) .

degree: كم سهم يطلع من
الرقم او الـ Vertic .

مكونه من شيئين اساسين :
1- Vertex .
2- edge.



(7, 8)
(8, 7)



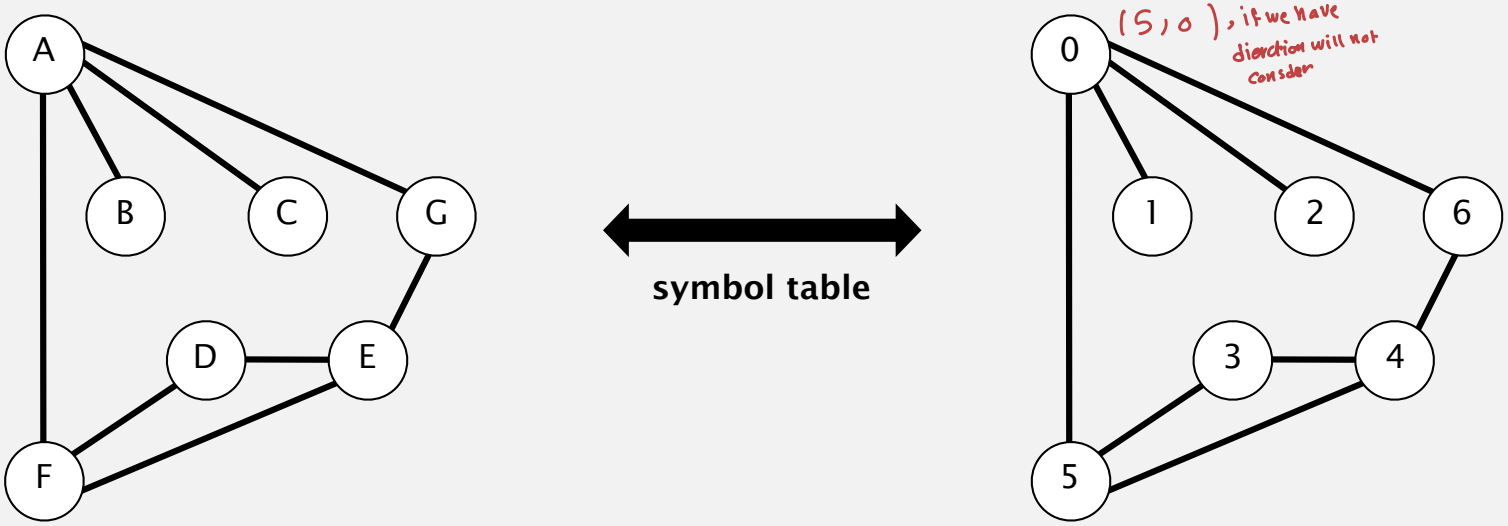
two drawings of the same graph

Caveat. Intuition can be misleading.

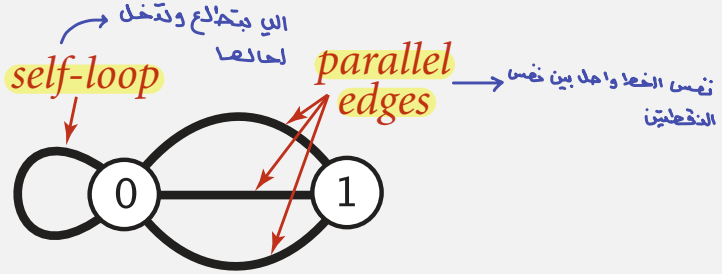
Graph representation

Vertex representation.

- This lecture: use integers between 0 and $V - 1$.
- Applications: convert between names and integers with symbol table.



Anomalies.



Graph API

public class Graph	
Graph(int V)	*create an empty graph with V vertices*
Graph(In in)	*create a graph from input stream*
void addEdge(int v, int w)	*add an edge v-w*
Iterable<Integer> adj(int v)	*vertices adjacent to v*
int V()	*number of vertices*
int E()	*number of edges*

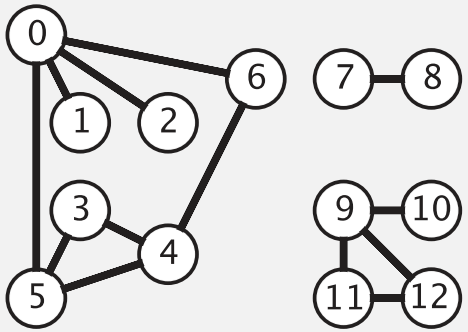
```
// degree of vertex v in graph G
public static int degree(Graph G, int v)
{
    int degree = 0;
    for (int w : G.adj(v))
       degree++;
    return degree;
}
```

Graph API: sample client

Graph input format.

tinyG.txt

```
13
13
0 5
4 3
0 1
9 12
6 4
5 4
0 2
11 12
9 10
0 6
7 8
9 11
5 3
```



adjacent : العقده الى اقدر انتقل بالـ Vertex الثانيه خلالها.

ماذا اذا رجع الاقي من (5,0) لانه undirected.

لو كنت شاكه ايش نوع (by default): undirected الا اذا كتبت بالسؤال directed.

dynamic data

how many time edge : o(e)

```
% java Test tinyG.txt
0-6
0-2
0-1
0-5
1-0
2-0
3-5
3-4
⋮
12-11
12-9
```

directed

```
In in = new In(args[0]);
Graph G = new Graph(in);

for (int v = 0; v < G.V(); v++)
   for (int w : G.adj(v))
      StdOut.println(v + "-" + w);
```

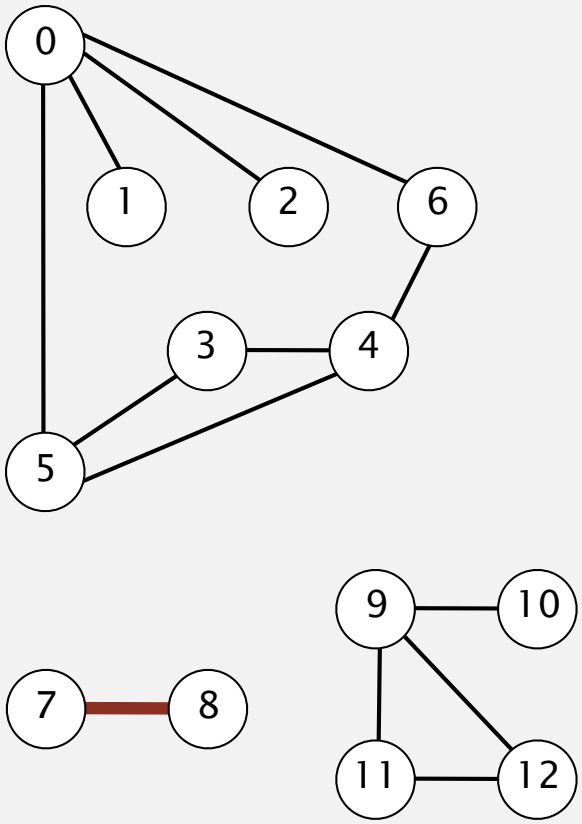
read graph from input stream

print out each edge (twice)

Graph representation: set of edges

Maintain a list of the edges (linked list or array).

degree: كم خط طالع منها



اللاين اجنس لان كل واحد مكرر

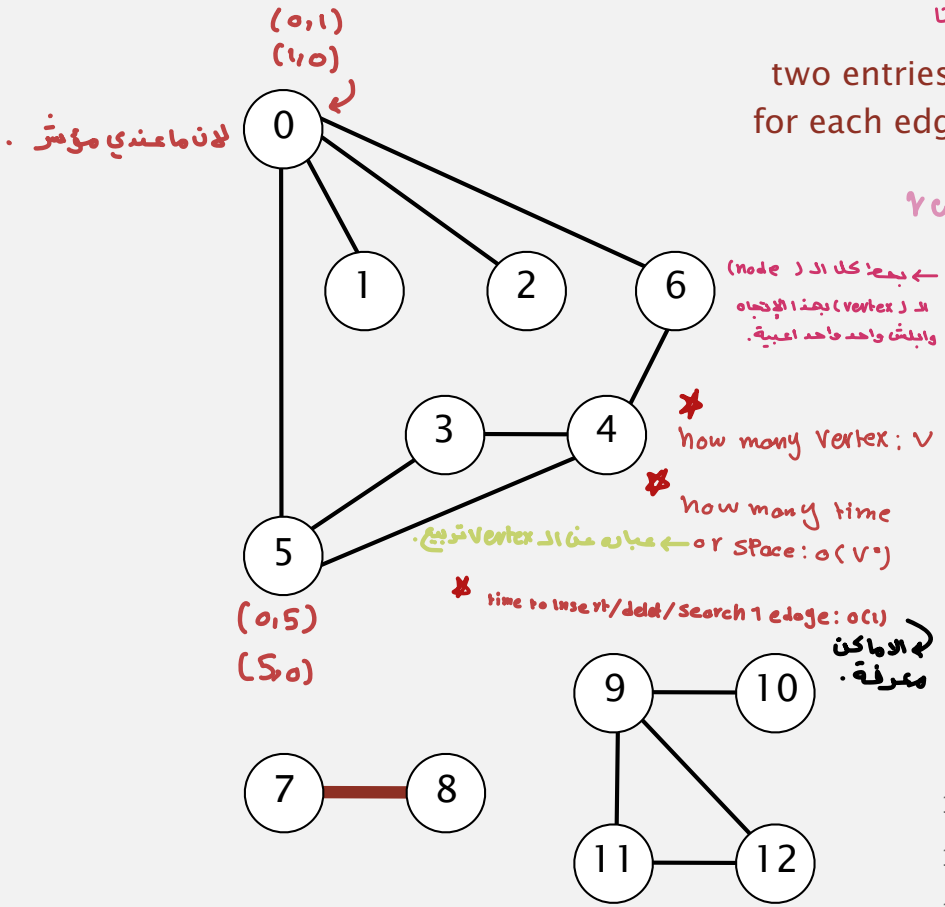
0	1
0	2
0	5
0	6
3	4
3	5
4	5
4	6
7	8
9	10
9	11
9	12
11	12

Q. How long to iterate over vertices adjacent to v ?

Graph representation: adjacency matrix

Maintain a two-dimensional V-by-V boolean array;

for each edge v–w in graph: `adj[v][w] = adj[w][v] = true`.



two entries for each edge

	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	1	1	0	0	1	1	0	0	0	0	0	0
1	1	0	0	0	0	0	0	0	0	0	0	0	0
2	1	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	1	1	0	0	0	0	0	0	0
4	0	0	0	1	0	1	1	0	0	0	0	0	0
5	1	0	0	1	1	0	0	0	0	0	0	0	0
6	1	0	0	0	1	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	1	0	0	0	0
8	0	0	0	0	0	0	0	1	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	1	1	1
10	0	0	0	0	0	0	0	0	0	1	0	0	0
11	0	0	0	0	0	0	0	0	0	1	0	0	1
12	0	0	0	0	0	0	0	0	0	1	0	1	0

how many vertex : V

how many time or space : O(V²)

time to insert/delete/search 1 edge : O(1)

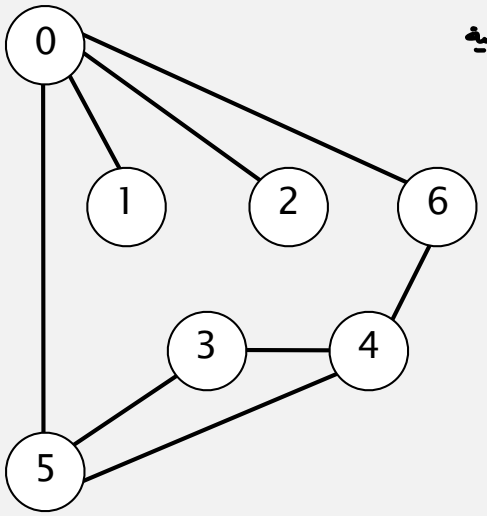
Symmetric undirected

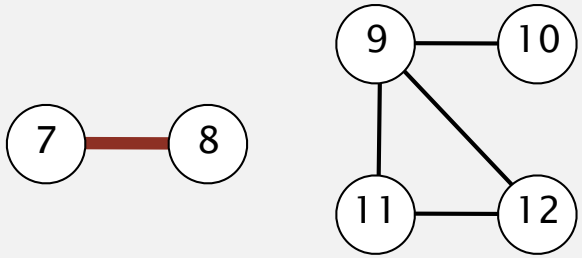
Q. How long to iterate over vertices adjacent to v ? O(V)

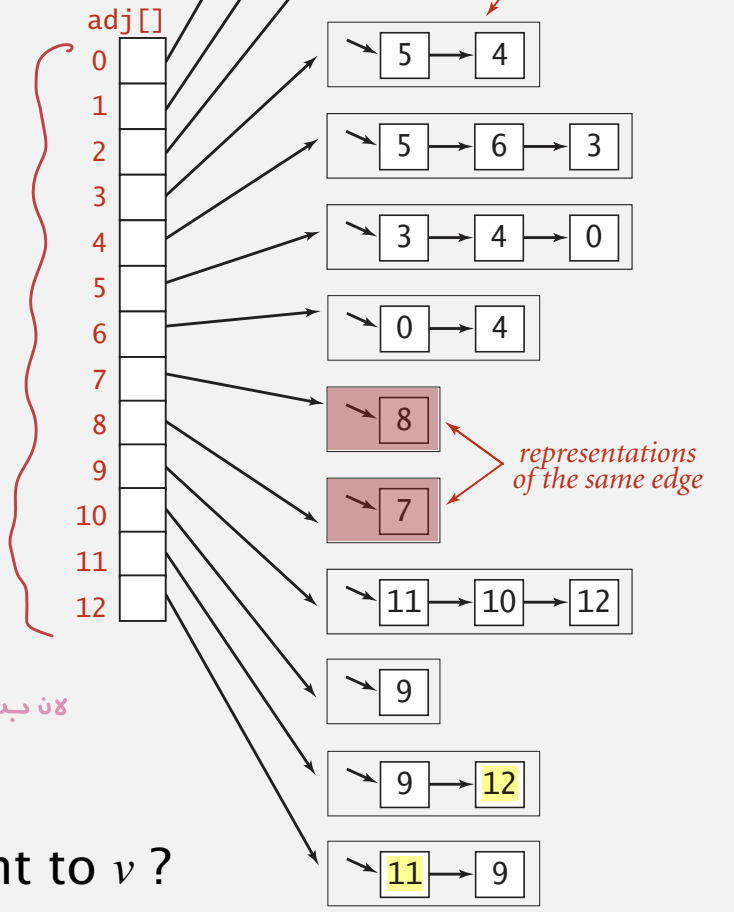
How long to iterate over all vertices : O(V²)

Graph representation: adjacency lists

Maintain vertex-indexed array of lists.







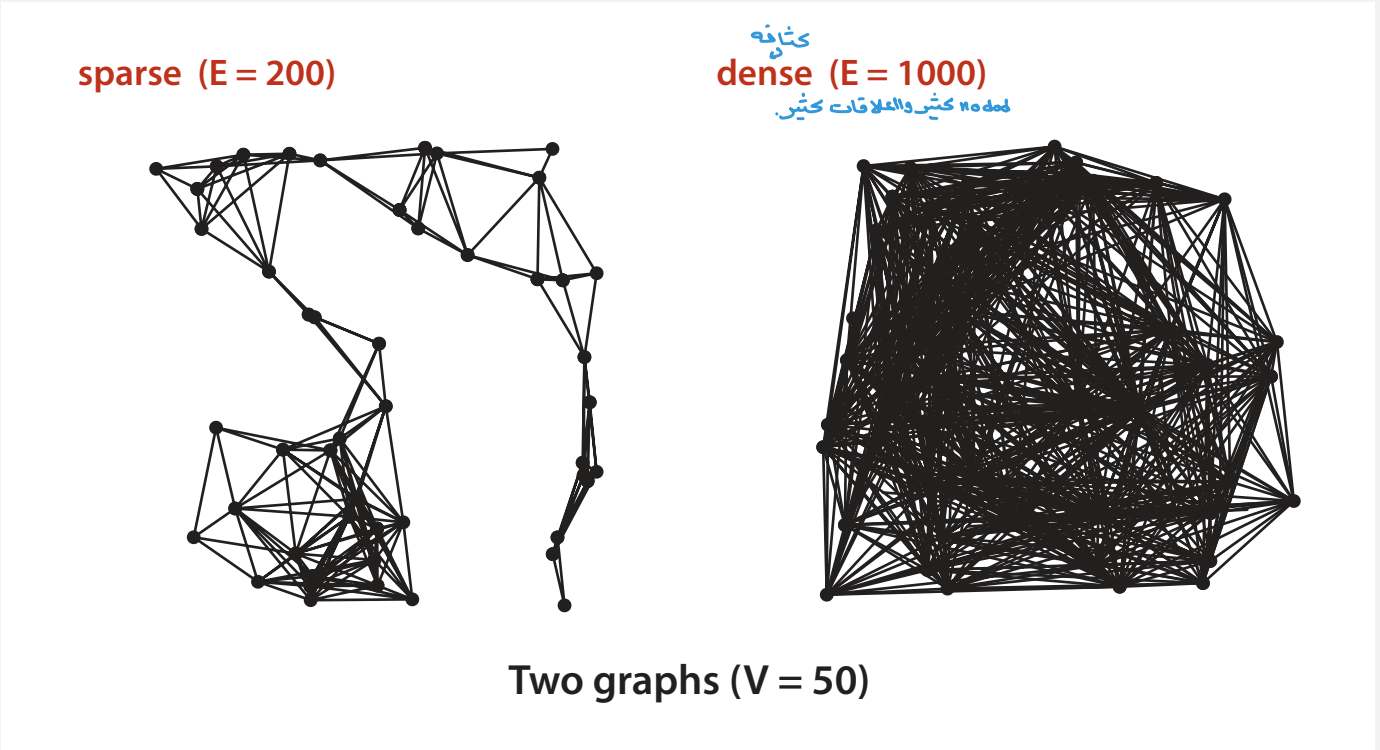
Q. How long to iterate over vertices adjacent to v ?

Graph representations

In practice. Use adjacency-lists representation.

- Algorithms based on iterating over vertices adjacent to v.
- Real-world graphs tend to be sparse.

huge number of vertices,
small average vertex degree



Two graphs (V = 50)

Graph representations

In practice. Use adjacency-lists representation.

- Algorithms based on iterating over vertices adjacent to v.
- Real-world graphs tend to be sparse.

huge number of vertices,
small average vertex degree

representation	space	add edge	edge between v and w?	iterate over vertices adjacent to v?
list of edges	E	1	$O(E)$	$O(E)$
adjacency matrix	V^2	1 *	1	$O(V)$
adjacency lists	$E + V$	1	$degree(v)$	$degree(v)$

* disallows parallel edges

Adjacency-list graph representation: Java implementation

```
public class Graph
{
   private final int V;
   private Bag<Integer>[] adj;

   public Graph(int V)
   {
      this.V = V;
      adj = (Bag<Integer>[]) new Bag[V];
      for (int v = 0; v < V; v++)
         adj[v] = new Bag<Integer>();
   }

   public void addEdge(int v, int w)
   {
      adj[v].add(w);
      adj[w].add(v);
   }

   public Iterable<Integer> adj(int v)
   {  return adj[v];  }
}
```

adjacency lists
(using Bag data type)

create empty graph
with V vertices

add edge v-w
(parallel edges and
self-loops allowed)

iterator for vertices adjacent to v



http://algs4.cs.princeton.edu

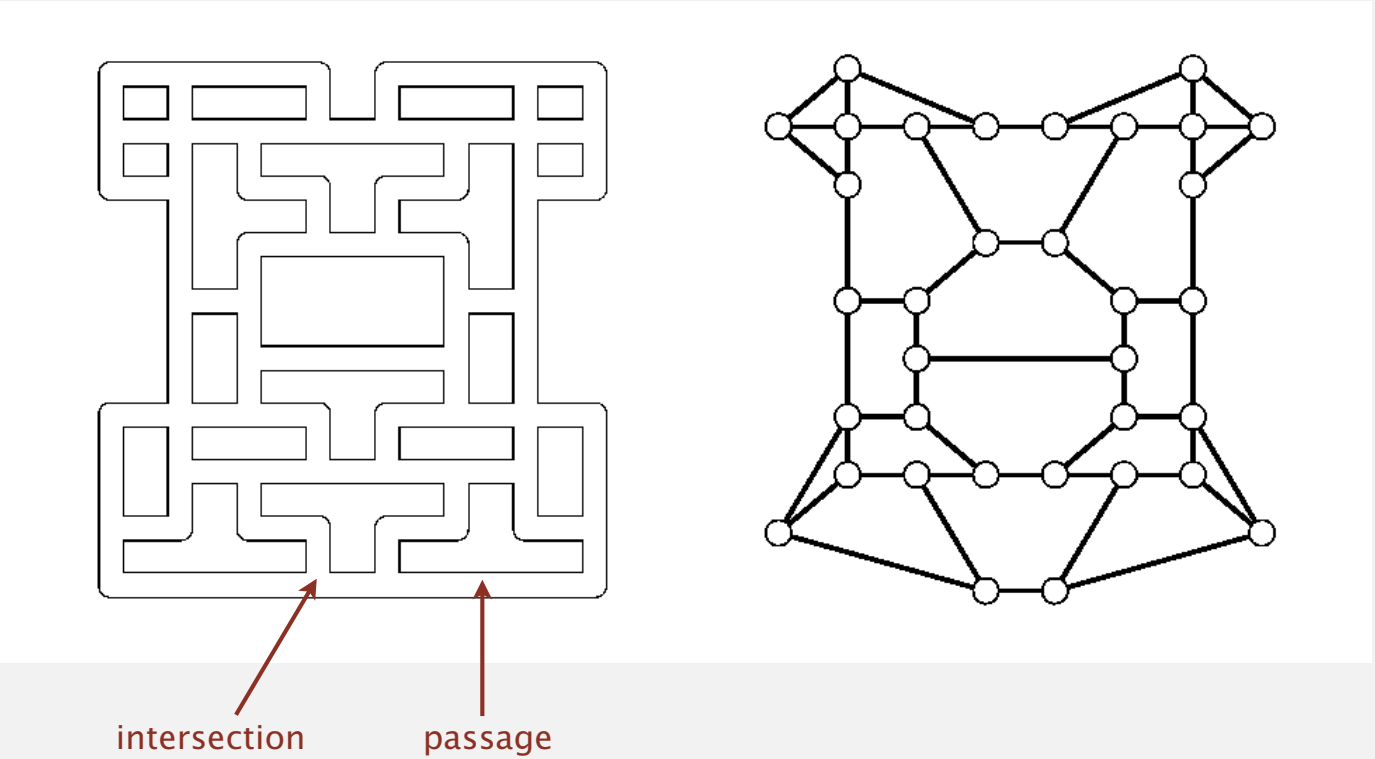
4.1 Undirected Graphs

- *introduction*
- *graph API*
- *depth-first search*
- *breadth-first search*
- *connected components*
- *challenges*

Maze exploration

Maze graph. back track

- Vertex = intersection.
- Edge = passage.

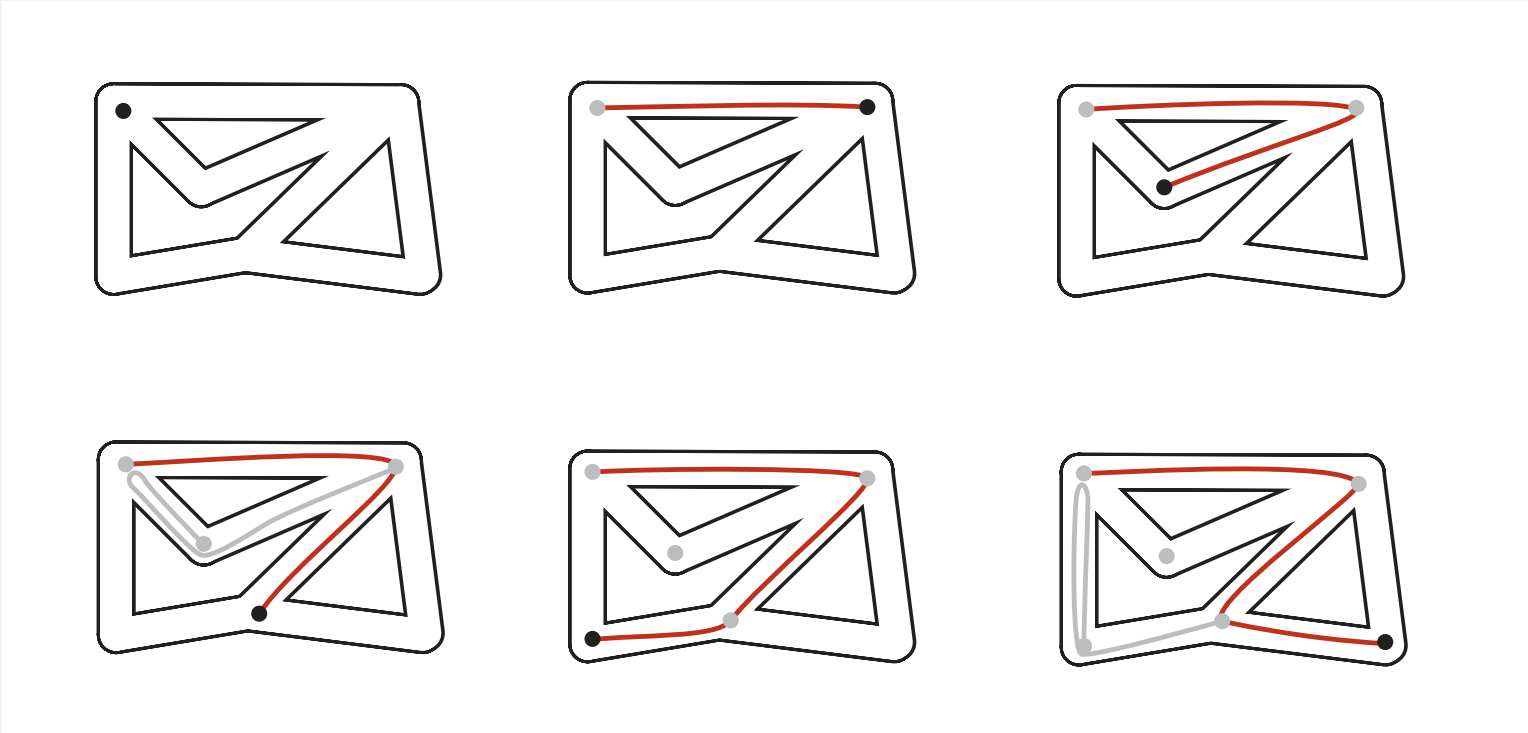


Goal. Explore every intersection in the maze.

Trémaux maze exploration

Algorithm.

- Unroll a ball of string behind you.
- Mark each visited intersection and each visited passage.
- Retrace steps when no unvisited options.



Trémaux maze exploration

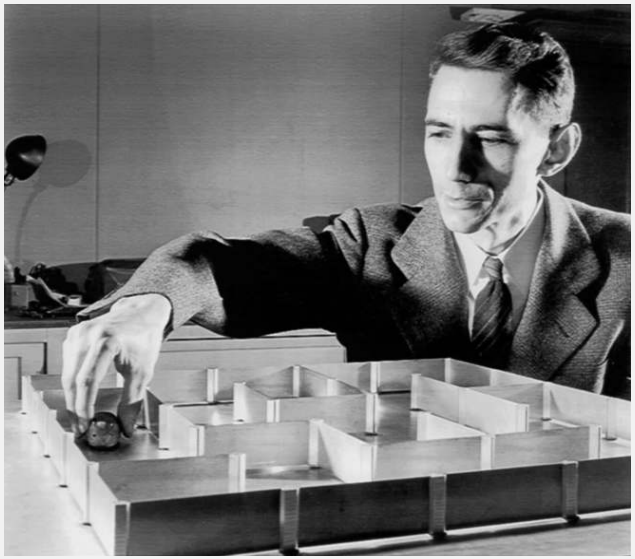
Algorithm.

- Unroll a ball of string behind you.
- Mark each visited intersection and each visited passage.
- Retrace steps when no unvisited options.

First use? Theseus entered Labyrinth to kill the monstrous Minotaur; Ariadne instructed Theseus to use a ball of string to find his way back out.

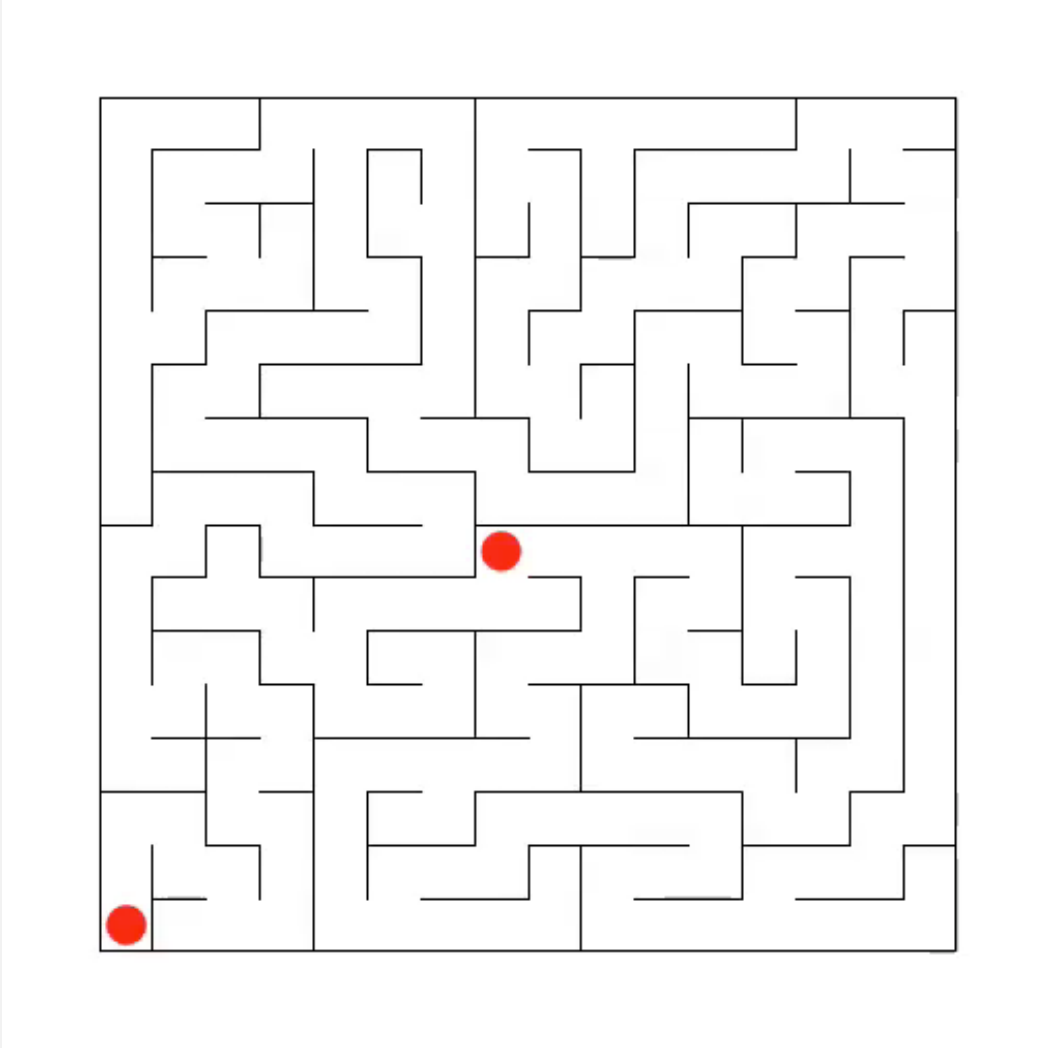


The Labyrinth (with Minotaur)

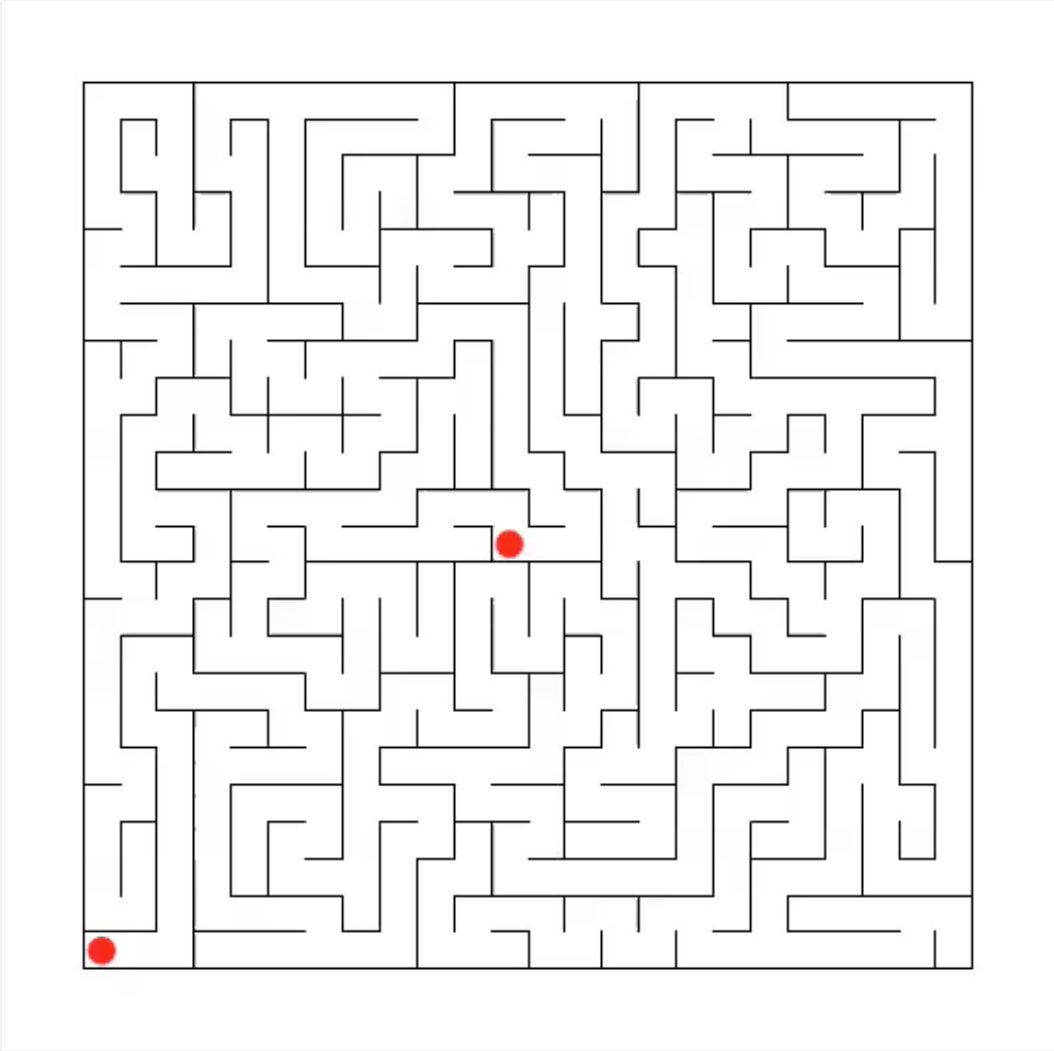


Claude Shannon (with Theseus mouse)

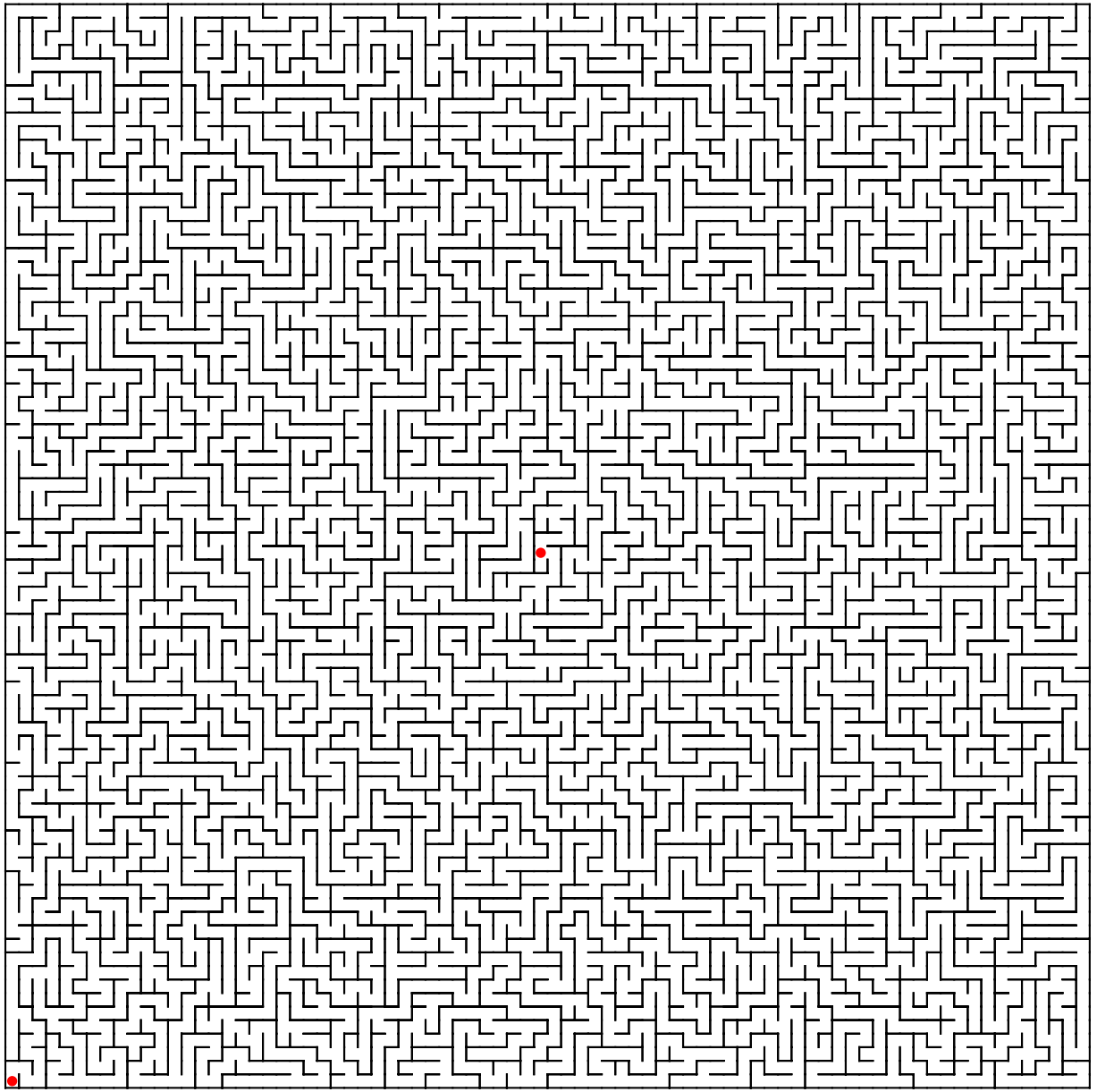
Maze exploration: easy



Maze exploration: medium



Maze exploration: challenge for the bored



Depth-first search → stack تستخدم

Goal. Systematically traverse a graph.

Idea. Mimic maze exploration. ← function-call stack acts as ball of string

بمر بكل النقاط عندي .

DFS (to visit a vertex v)

Mark v as visited.

كل الطرق متاحه

Recursively visit all unmarked vertices w adjacent to v.

بروح بختار النقطه اللي قال لي ابدا منها بعمل انها (visited)

Typical applications.

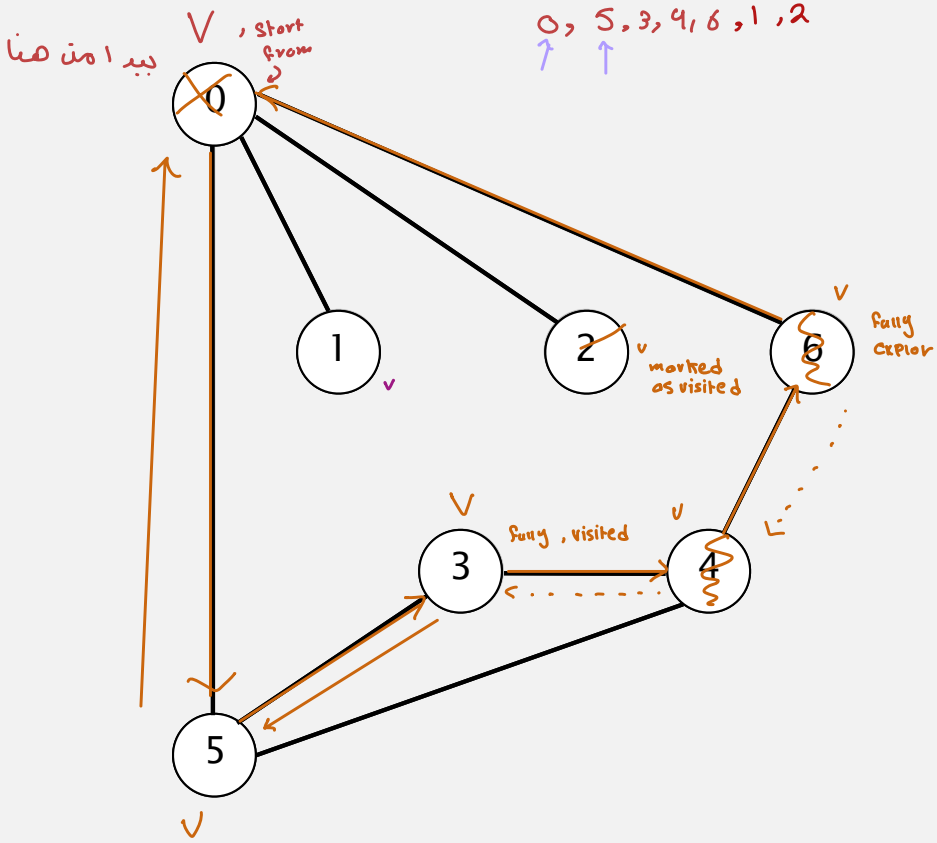
- Find all vertices connected to a given source vertex.
- Find a path between two vertices.

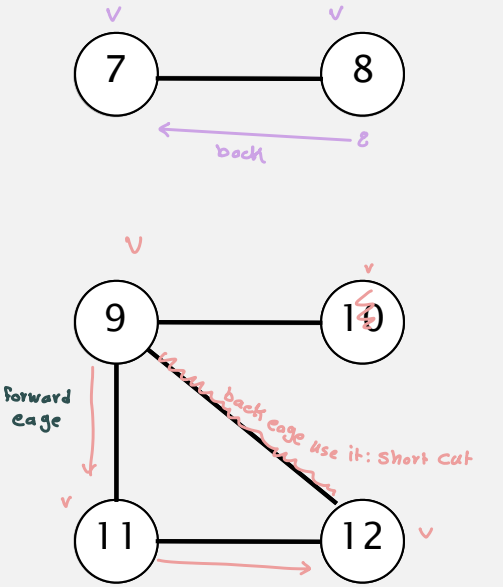
Design challenge. How to implement?

Depth-first search demo

To visit a vertex v :

- Mark vertex v as visited.
- Recursively visit all unmarked vertices adjacent to v.





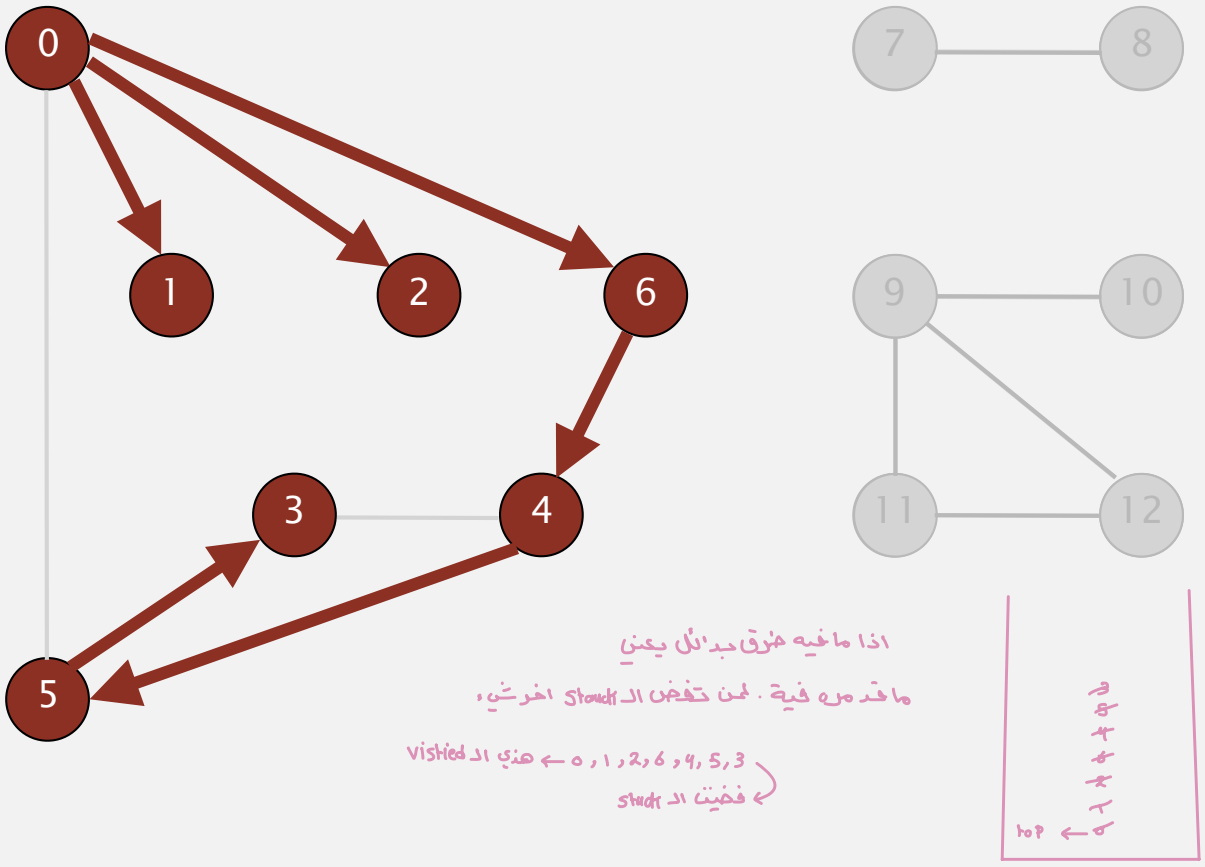


graph G

Depth-first search demo

To visit a vertex v :

- Mark vertex v as visited.
- Recursively visit all unmarked vertices adjacent to v.



v	marked[]	edgeTo[]
0	T	–
1	T	0
2	T	0
3	T	5
4	T	6
5	T	4
6	T	0
7	F	–
8	F	–
9	F	–
10	F	–
11	F	–
12	F	–

vertices reachable from 0

Design pattern for graph processing

Design pattern. Decouple graph data type from graph processing.

- Create a `Graph` object.
- Pass the `Graph` to a graph-processing routine.
- Query the graph-processing routine for information.

public class Paths		
	`Paths(Graph G, int s)`	*find paths in G from source s*
`boolean`	`hasPathTo(int v)`	*is there a path from s to v?*
`Iterable<Integer>`	`pathTo(int v)`	*path from s to v; null if no such path*

```
Paths paths = new Paths(G, s);
for (int v = 0; v < G.V(); v++)
   if (paths.hasPathTo(v))
      StdOut.println(v);
```

print all vertices connected to s

Depth-first search: data structures

To visit a vertex v :

- Mark vertex v as visited.
- Recursively visit all unmarked vertices adjacent to v.

Data structures.

- Boolean array `marked[]` to mark visited vertices.
- Integer array `edgeTo[]` to keep track of paths.

  `(edgeTo[w] == v)` means that edge `v-w` taken to visit `w` for first time
- Function-call stack for recursion.

Depth-first search: Java implementation

```
public class DepthFirstPaths
{
   private boolean[] marked;
   private int[] edgeTo;
   private int s;

   public DepthFirstPaths(Graph G, int s)
   {
      ...
      dfs(G, s);
   }

   private void dfs(Graph G, int v)
   {
      marked[v] = true;
      for (int w : G.adj(v))
         if (!marked[w])
         {
            dfs(G, w);
            edgeTo[w] = v;
         }
   }
}
```

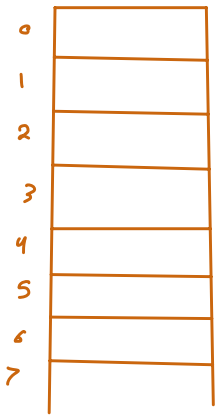
marked[v] = true if v connected to s

edgeTo[v] = previous vertex on path from s to v

initialize data structures

find vertices connected to s

recursive DFS does the work



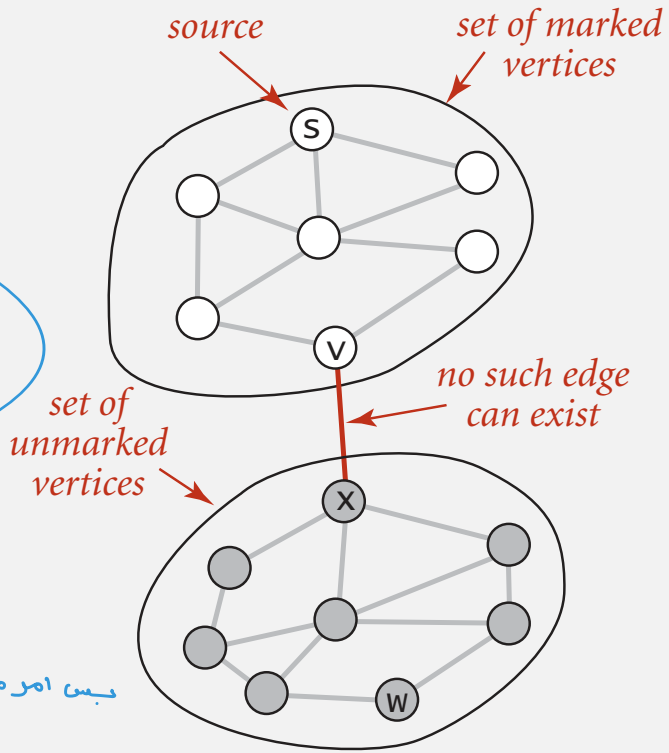
Depth-first search: properties

Proposition. DFS marks all vertices connected to s in time proportional to the sum of their degrees (plus time to initialize the `marked[]` array).

Pf. [correctness]

- If w marked, then w connected to s (why?)
- If w connected to s, then w marked.
  (if w unmarked, then consider last edge on a path from s to w that goes from a marked vertex to an unmarked one).

في ما معناه : اذا بس خلصت وكان معنى وحده (mark) فهي مربوطة بطريقه معينة مع الـ (s) اللي بديت فيها . واذا معنى وحده (unmarked) معناته ما فيه (edge) يوصل منها .



Pf. [running time]

Each vertex connected to s is visited once. → بس امر مره وحدة

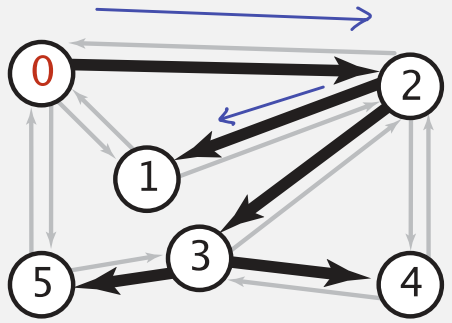
Depth-first search: properties

Proposition. After DFS, can check if vertex v is connected to s in constant time and can find v–s path (if one exists) in time proportional to its length.

Pf. edgeTo[] is parent-link representation of a tree rooted at vertex s.

```
public boolean hasPathTo(int v)
{  return marked[v];  }

public Iterable<Integer> pathTo(int v)
{
   if (!hasPathTo(v)) return null;
   Stack<Integer> path = new Stack<Integer>();
   for (int x = v; x != s; x = edgeTo[x])
      path.push(x);
   path.push(s);
   return path;
}
```



edgeTo[]

0	
1	2
2	0
3	2
4	3
5	3

Depth-first search application: flood fill

Challenge. Flood fill (Photoshop magic wand).

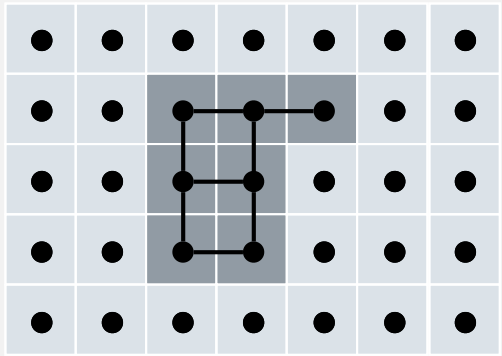
Assumptions. Picture has millions to billions of pixels.



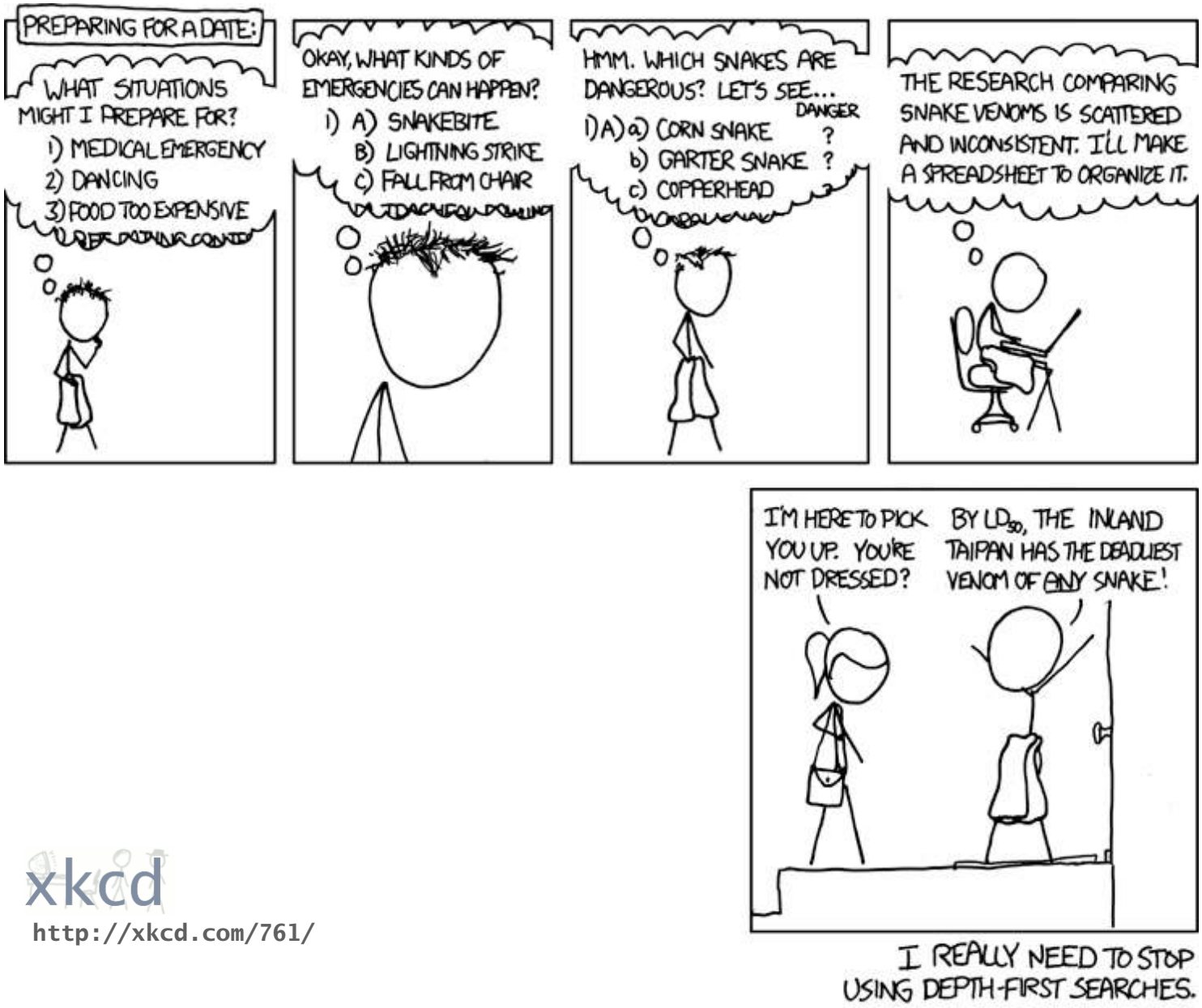


Solution. Build a grid graph (implicitly).

- Vertex: pixel.
- Edge: between two adjacent gray pixels.
- Blob: all pixels connected to given pixel.



Depth-first search application: preparing for a date



xkcd

http://xkcd.com/761/



http://algs4.cs.princeton.edu

4.1 Undirected Graphs

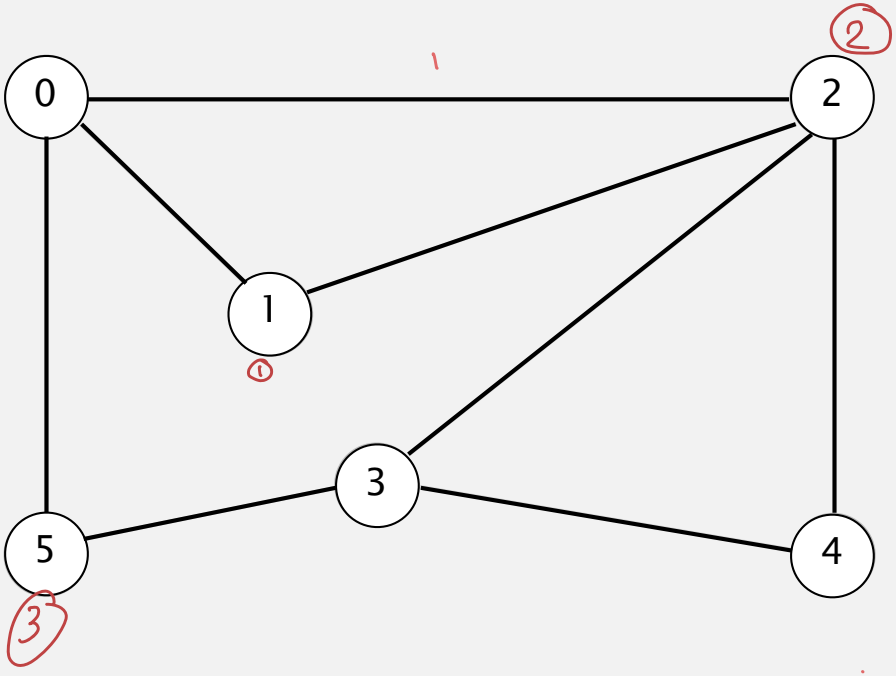
- *introduction*
- *graph API*
- *depth-first search*
- *breadth-first search*
- *connected components*
- *challenges*

Sorted path ⟶

Breadth-first search demo

Repeat until queue is empty:

- Remove vertex v from queue.
- Add to queue all unmarked vertices adjacent to v and mark them.



graph G

tinyCG.txt

V
6
E
8
0 5
2 4
2 3
1 2
0 1
3 4
3 5
0 2

use a Queue

Q

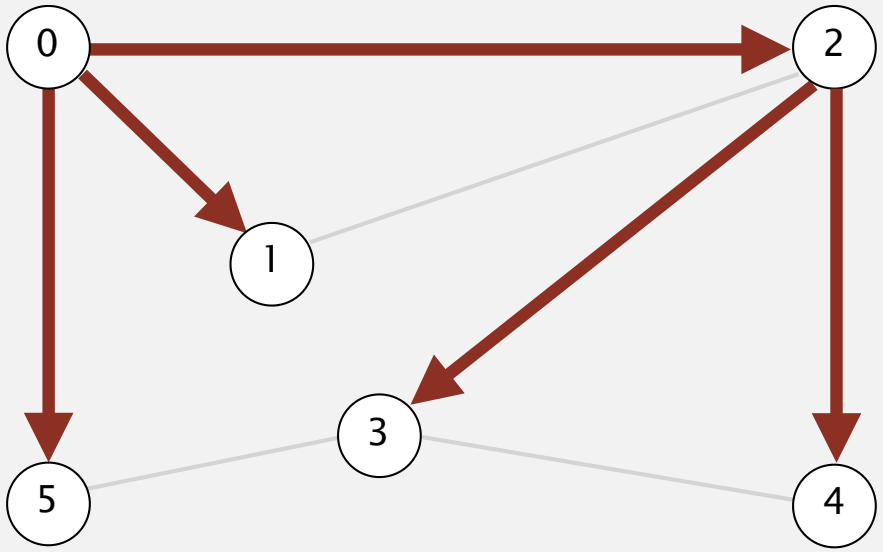


0, 1, 2, 5, 3, 4

Breadth-first search demo

Repeat until queue is empty:

- Remove vertex v from queue.
- Add to queue all unmarked vertices adjacent to v and mark them.



v	edgeTo[]	distTo[]
0	–	0
1	0	1
2	0	1
3	2	2
4	2	2
5	0	1

runtime: $O(V+E)$

done

Breadth-first search

Repeat until queue is empty:

- Remove vertex v from queue.
- Add to queue all unmarked vertices adjacent to v and mark them.

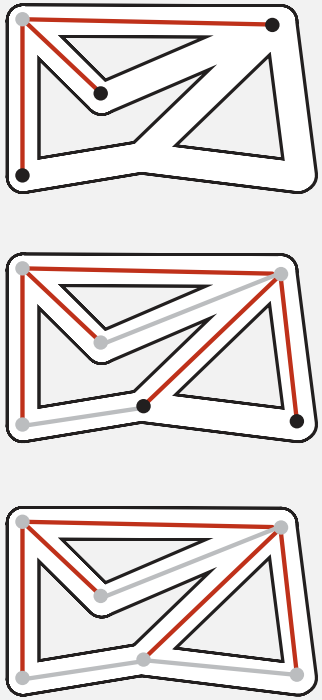
BFS (from source vertex s)

Put s onto a FIFO queue, and mark s as visited.

Repeat until the queue is empty:

- remove the least recently added vertex v

- add each of v's unvisited neighbors to the queue, and mark them as visited.



Sample: ماصي مكررة

not sample: مكررة

Breadth-first search: Java implementation

```
public class BreadthFirstPaths
{
   private boolean[] marked;
   private int[] edgeTo;
   private int[] distTo;

   …

   private void bfs(Graph G, int s) {
      Queue<Integer> q = new Queue<Integer>();
      q.enqueue(s);
      marked[s] = true;
      distTo[s] = 0;

      while (!q.isEmpty()) {
         int v = q.dequeue();
         for (int w : G.adj(v)) {
            if (!marked[w]) {
               q.enqueue(w);
               marked[w] = true;
               edgeTo[w] = v;
               distTo[w] = distTo[v] + 1;
            }
         }
      }
   }
}
```

initialize FIFO queue of vertices to explore

found new vertex w via edge v-w

Breadth-first search properties

Q. In which order does BFS examine vertices?

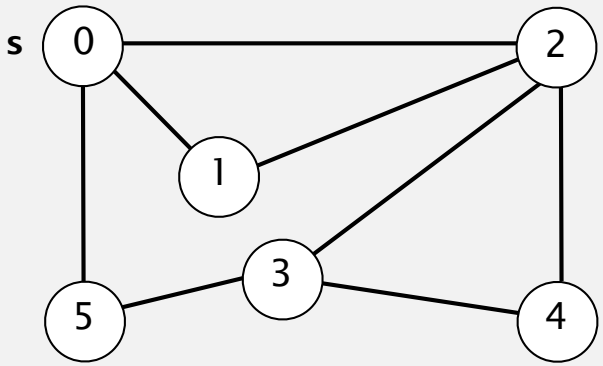
A. Increasing distance (number of edges) from s.

ممكن نعتمد على الحروف .

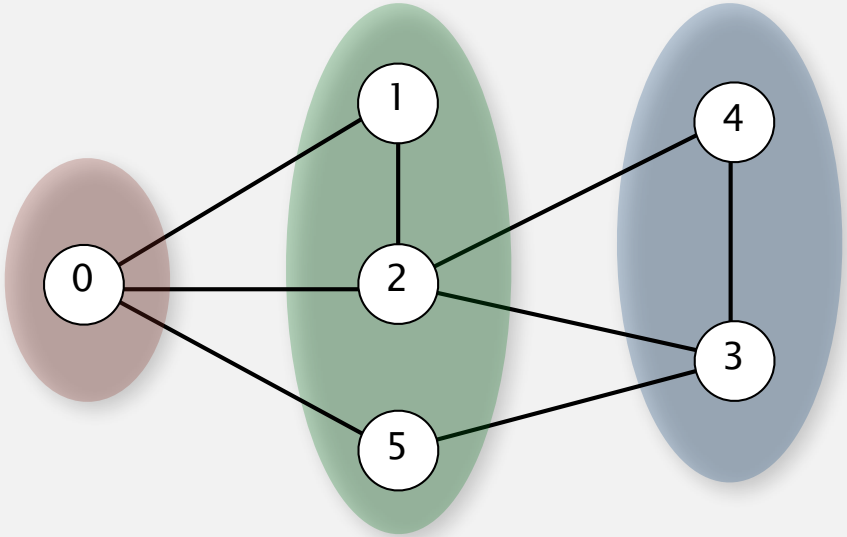
queue always consists of ≥ 0 vertices of distance k from s, followed by ≥ 0 vertices of distance $k+1$

Proposition. In any connected graph G, BFS computes shortest paths from s to all other vertices in time proportional to $E + V$.

مرات : استخدمي المسافات من الصغير للكبير : اصغر مسافه (1) , (2), (3)
ممكن استخدمي المسافات من الكبير للصغير: اكبر مسافه : (3) , (2) , (1)



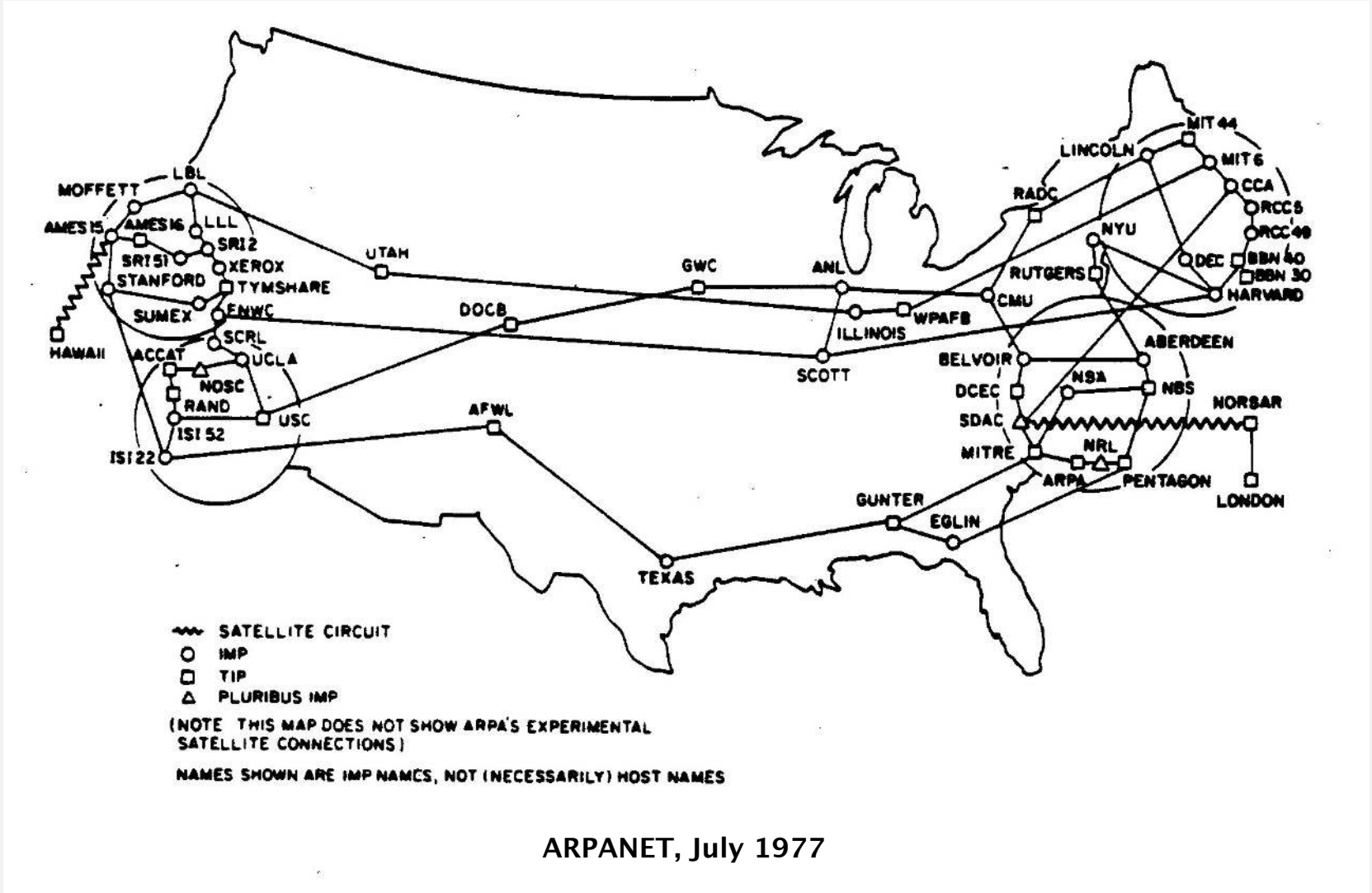
graph G



dist = 0 dist = 1 dist = 2

Breadth-first search application: routing

Fewest number of hops in a communication network.



ARPANET, July 1977

Breadth-first search application: Kevin Bacon numbers



http://oracleofbacon.org



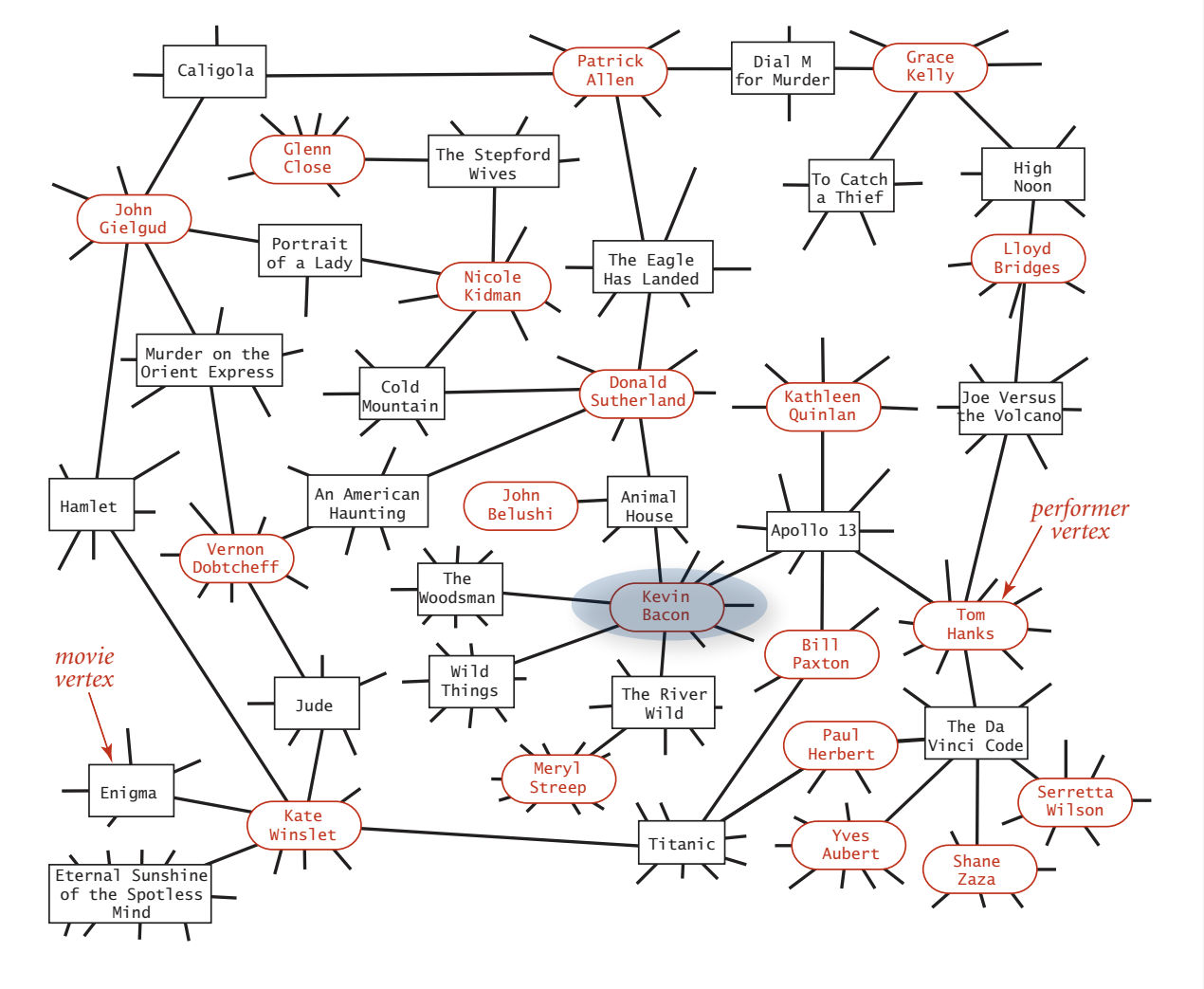
Endless Games board game



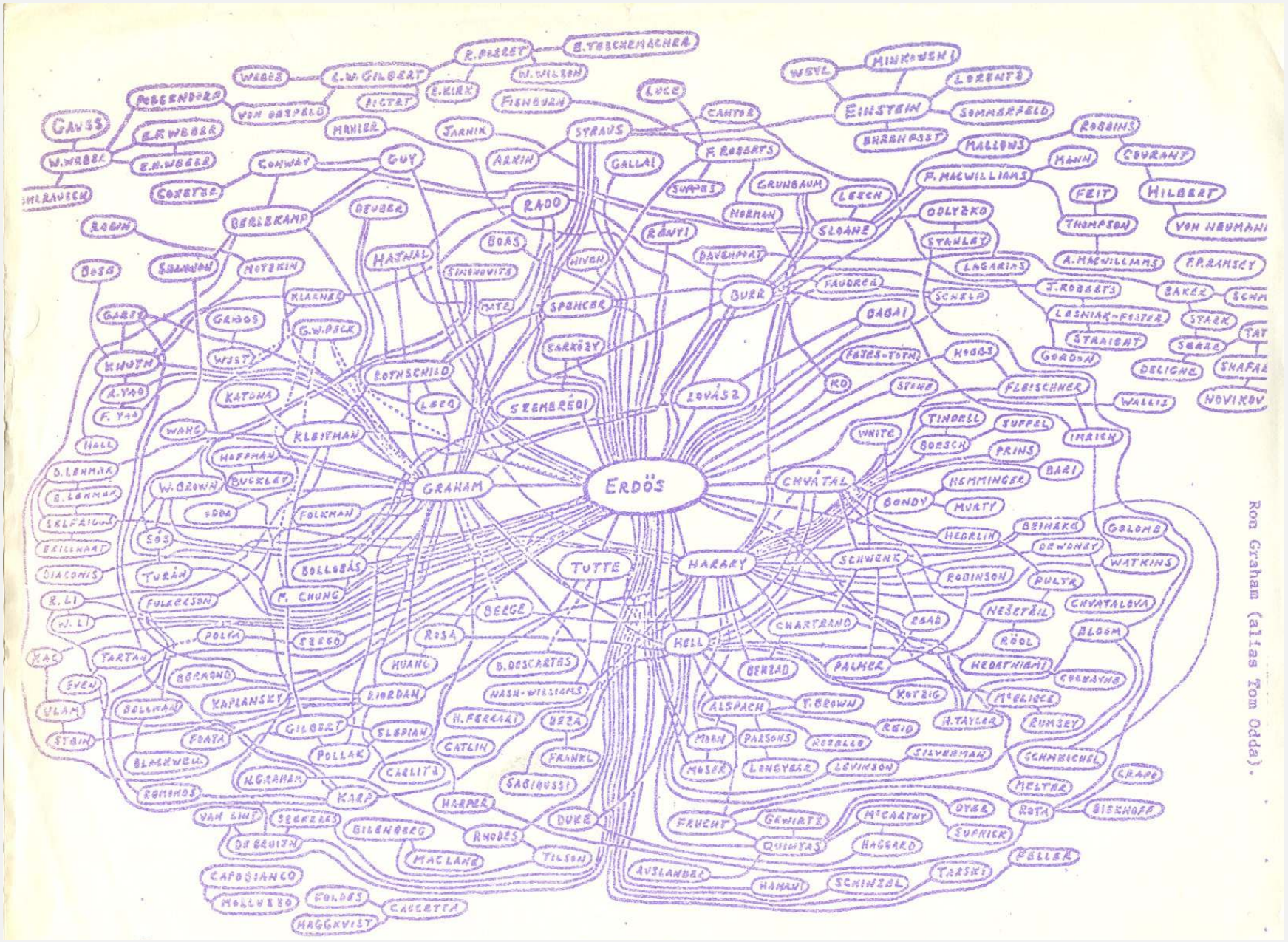
SixDegrees iPhone App

Kevin Bacon graph

- Include one vertex for each performer and one for each movie.
- Connect a movie to all performers that appear in that movie.
- Compute shortest path from s = Kevin Bacon.



Breadth-first search application: Erdös numbers



hand-drawing of part of the Erdös graph by Ron Graham



http://algs4.cs.princeton.edu

4.1 Undirected Graphs

- *introduction*
- *graph API*
- *depth-first search*
- *breadth-first search*
- *connected components*
- *challenges*

Connectivity queries

Def. Vertices v and w are connected if there is a path between them.



Goal. Preprocess graph to answer queries of the form *is v connected to w?* in constant time.

public class CC	
CC(Graph G)	*find connected components in G*
boolean connected(int v, int w)	*are v and w connected?*
int count()	*number of connected components*
int id(int v)	*component identifier for v (between 0 and count() - 1)*

Union-Find? Not quite.

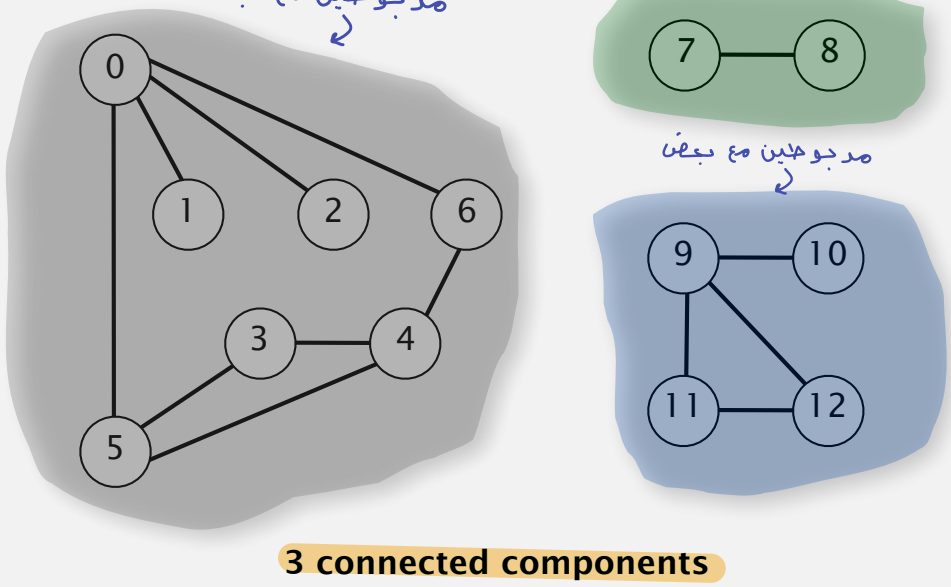
Depth-first search. Yes. [next few slides]

Connected components

The relation "is connected to" is an equivalence relation:

- Reflexive: v is connected to v.
- Symmetric: if v is connected to w, then w is connected to v.
- Transitive: if v connected to w and w connected to x, then v connected to x.

Def. A connected component is a maximal set of connected vertices.



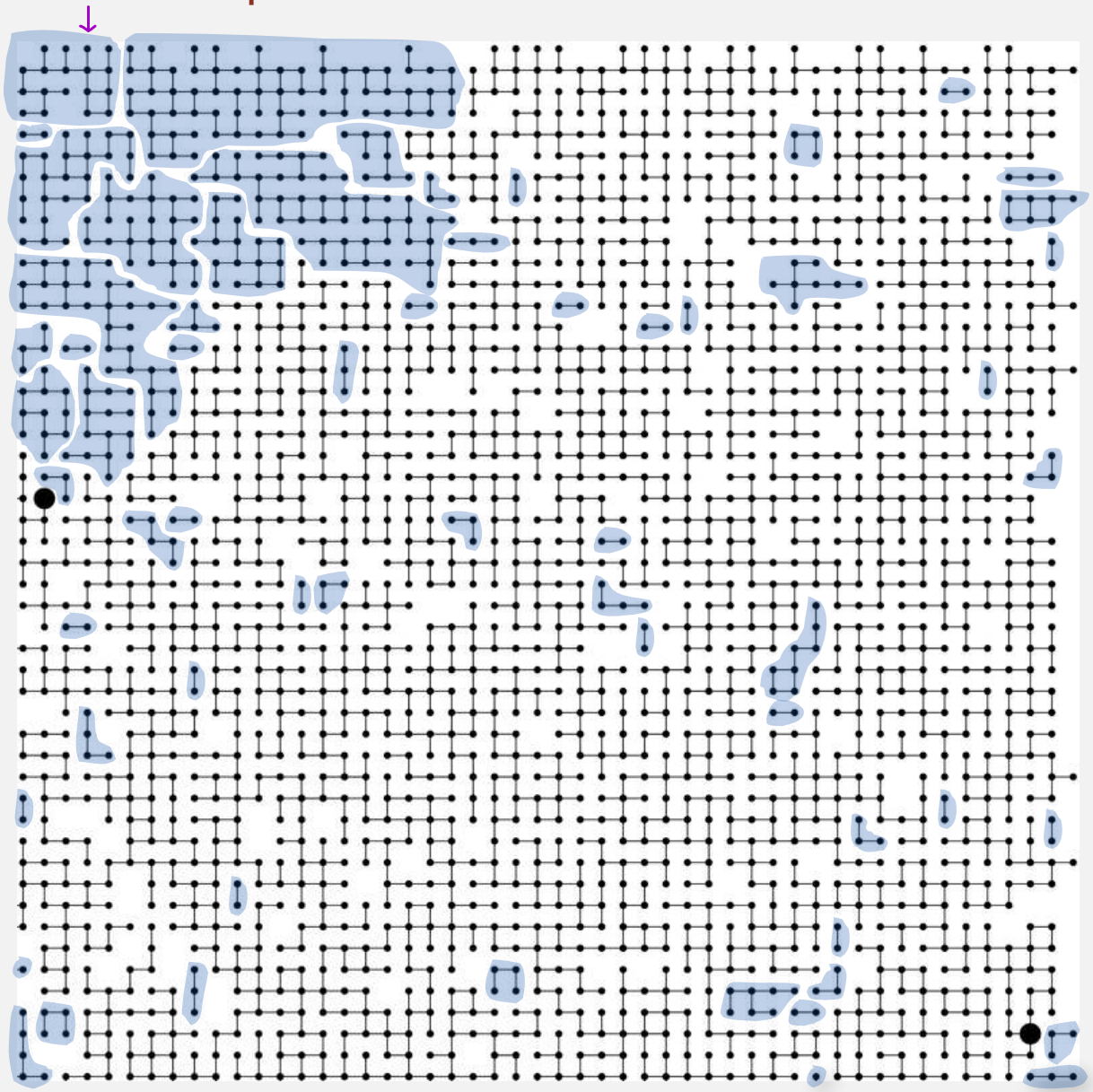
3 connected components

v	id[]
0	0
1	0
2	0
3	0
4	0
5	0
6	0
7	1
8	1
9	2
10	2
11	2
12	2

Remark. Given connected components, can answer queries in constant time.

Connected components

Def. A connected component is a maximal set of connected vertices.



63 connected components

Connected components

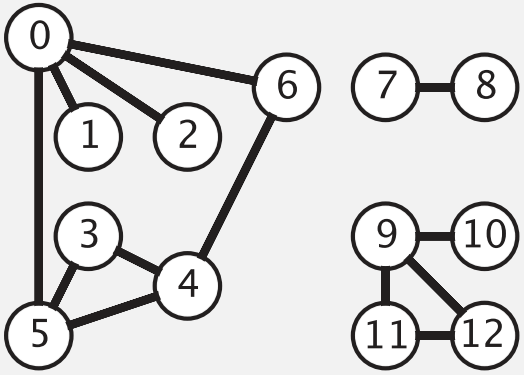
Goal. Partition vertices into connected components.

Connected components

Initialize all vertices v as unmarked.

يعني راح ابدا من الاصفر وبشوف
انه بيوصل لكل عنصر وبروح ادور على ثاني
ماهار د (Terse) وبشوف اذ (Visited) الي مدبوجين
فيه.

For each unmarked vertex v, run DFS to identify all vertices discovered as part of the same component.



tinyG.txt

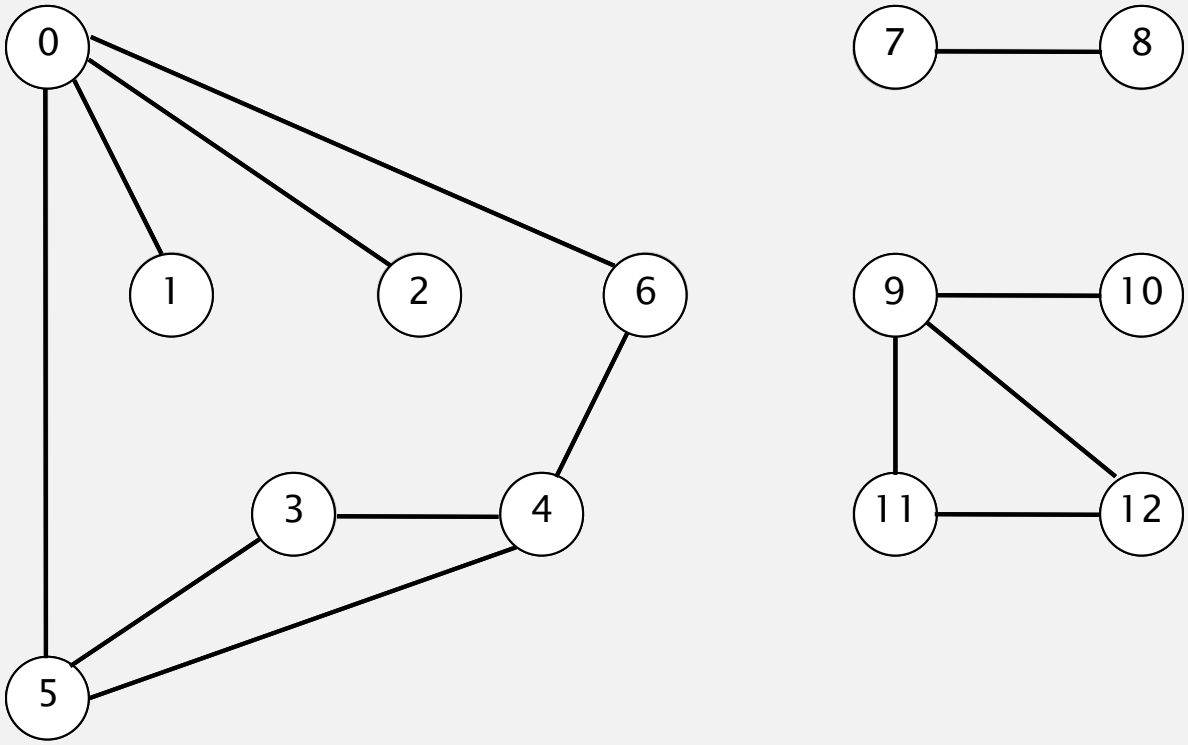
```
V → 13
    13 ← E
0  5
4  3
0  1
9  12
6  4
5  4
0  2
11  12
9  10
0  6
7  8
9  11
5  3
```

Connected components demo

To visit a vertex v :



- Mark vertex v as visited.
- Recursively visit all unmarked vertices adjacent to v.



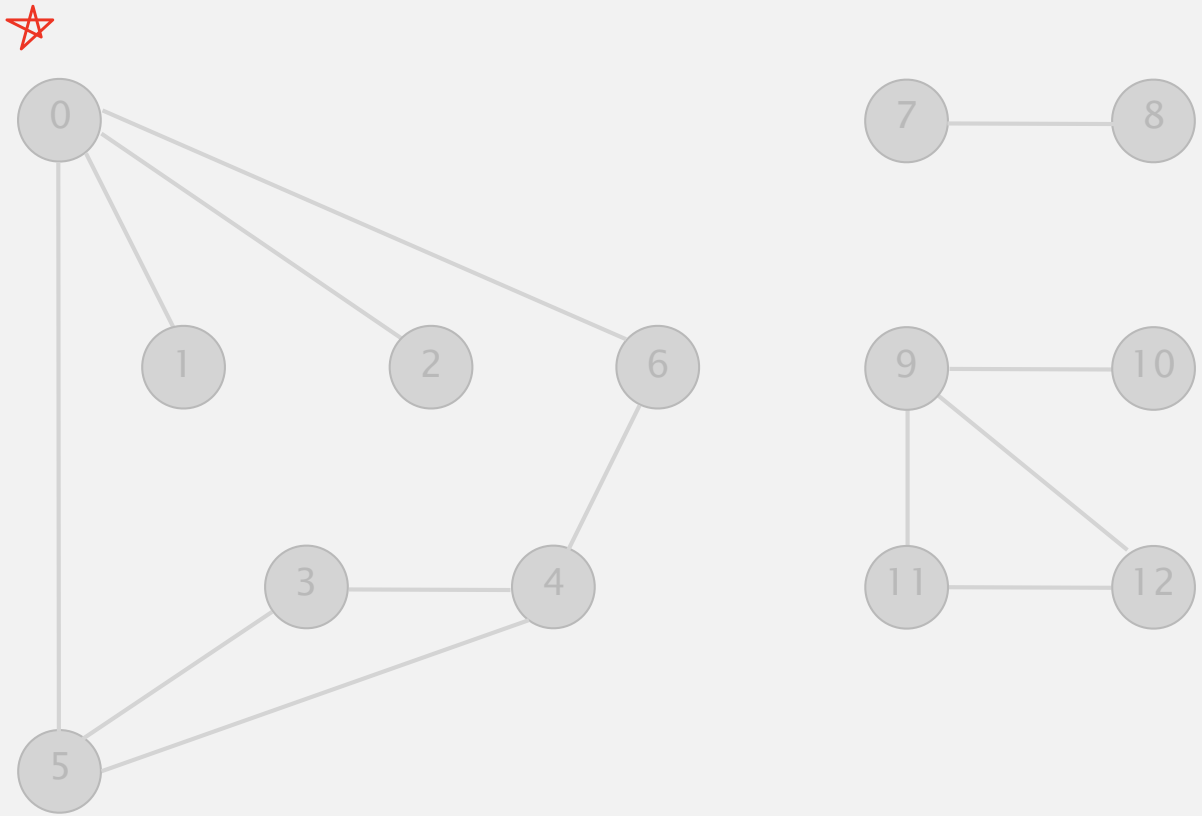
v	marked[]	id[]
0	F	–
1	F	–
2	F	–
3	F	–
4	F	–
5	F	–
6	F	–
7	F	–
8	F	–
9	F	–
10	F	–
11	F	–
12	F	–

graph G

Connected components demo

To visit a vertex v :

- Mark vertex v as visited.
- Recursively visit all unmarked vertices adjacent to v.



v	marked[]	id[]
0	T	0
1	T	0
2	T	0
3	T	0
4	T	0
5	T	0
6	T	0
7	T	1
8	T	1
9	T	2
10	T	2
11	T	2
12	T	2

done

Finding connected components with DFS

```
public class CC
{
   private boolean[] marked;
   private int[] id;
   private int count;

   public CC(Graph G)
   {
      marked = new boolean[G.V()];
      id = new int[G.V()];
      for (int v = 0; v < G.V(); v++)
      {
         if (!marked[v])
         {
            dfs(G, v);
            count++;
         }
      }
   }

   public int count()
   public int id(int v)
   public boolean connected(int v, int w)
   private void dfs(Graph G, int v)
}
```

id[v] = id of component containing v

number of components

run DFS from one vertex in each component

see next slide

Finding connected components with DFS (continued)

```
public int count()
{  return count;  }

public int id(int v)
{  return id[v];  }

public boolean connected(int v, int w)
{ return id[v] == id[w];  }


private void dfs(Graph G, int v)
{
   marked[v] = true;
   id[v] = count;
   for (int w : G.adj(v))
      if (!marked[w])
         dfs(G, w);
}
```

number of components

id of component containing v

v and w connected iff same id

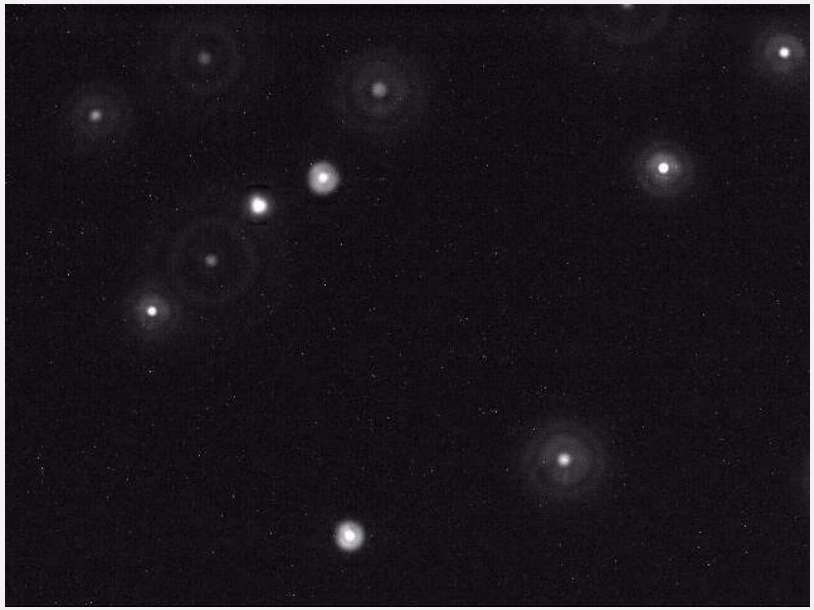
all vertices discovered in
same call of dfs have same id

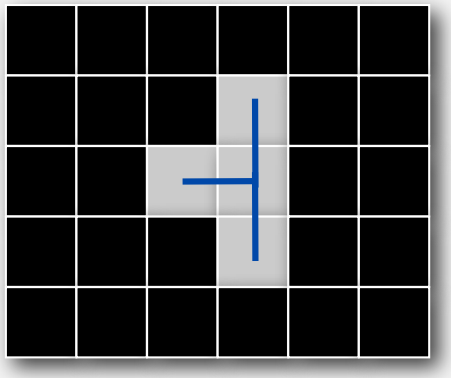
Connected components application: particle detection

Particle detection. Given grayscale image of particles, identify "blobs."

- Vertex: pixel.
- Edge: between two adjacent pixels with grayscale value ≥ 70.
- Blob: connected component of 20-30 pixels.

black = 0
white = 255





Particle tracking. Track moving particles over time.

Graph traversal summary

BFS and DFS enables efficient solution of many (but not all) graph problems.

problem	BFS	DFS	time
path between s and t	✔	✔	$E + V$
shortest path between s and t	✔		$E + V$
connected components	✔	✔	$E + V$
biconnected components		✔	$E + V$
cycle	✔	✔	$E + V$
Euler cycle		✔	$E + V$
Hamilton cycle			$2^{1.657\,V}$
bipartiteness	✔	✔	$E + V$
planarity		✔	$E + V$
graph isomorphism			$2^{c\sqrt{V \log V}}$